6.26

6.26 The streamlines in a certain incompressible, two-dimensional flow field are all concentric circles so that $v_r = 0$. Determine the stream function for **(a)** $v_\theta = Ar$ and for **(b)** $v_\theta = Ar^{-1}$, where A is a constant.

From the definition of the stream function,

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \qquad v_\theta = -\frac{\partial \psi}{\partial r} \qquad \text{(Eq. 6.42)}$$

so that with $v_r = 0$ it follows that $\frac{\partial \psi}{\partial \theta} = 0$

and therefore

$$\psi = f(r)$$

(a) For $v_\theta = Ar$

$$\frac{\partial \psi}{\partial r} = -Ar \qquad (1)$$

Integrate Eq. (1) with respect to r to obtain

$$\int d\psi = -\int Ar\,dr$$

or

$$\psi = -\frac{Ar^2}{2} + f_1(\theta)$$

However, since ψ is not a function of θ, it follows that

$$\psi = -\frac{Ar^2}{2} + C$$

where C is an arbitrary constant.

(b) Similarly, for $v_\theta = Ar^{-1}$

$$\int d\psi = -\int Ar^{-1}\,dr$$

or

$$\psi = -A \ln r + C$$