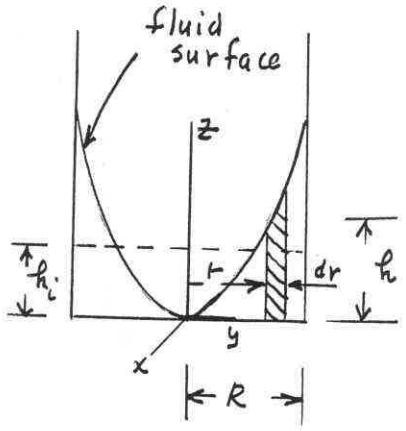
2.159

2.159 An open 1-m-diameter tank contains water at a depth of 0.7 m when at rest. As the tank is rotated about its vertical axis the center of the fluid surface is depressed. At what angular velocity will the bottom of the tank first be exposed? No water is spilled from the tank.



h_i ~ initial depth

Equation for surfaces of constant pressure (Eq. 2.32):

$$z = \frac{\omega^2 r^2}{2g} + \text{constant}$$

For free surface with $h = 0$ at $r = 0$,

$$h = \frac{\omega^2 r^2}{2g}$$

The volume of fluid in rotating tank is given by

$$\forall_f = \int_0^R 2\pi r h \, dr = \frac{2\pi\omega^2}{2g}\int_0^R r^3 dr = \frac{\pi\omega^2 R^4}{4g}$$

Since the initial volume, $\forall_i = \pi R^2 h_i$, must equal the final volume,

$$\forall_f = \forall_i$$

so that

$$\frac{\pi\omega^2 R^4}{4g} = \pi R^2 h_i$$

or

$$\omega = \sqrt{\frac{4g h_i}{R^2}} = \sqrt{\frac{4\left(9.81 \frac{m}{s^2}\right)(0.7m)}{(0.5m)^2}} = \underline{\underline{10.5 \frac{rad}{s}}}$$