3.8

3.8 An incompressible fluid flows steadily past a circular cylinder as shown in Fig. P3.8. The fluid velocity along the dividing streamline ($-\infty \le x \le -a$) is found to be $V = V_0(1 - a^2/x^2)$, where a is the radius of the cylinder and V_0 is the upstream velocity. **(a)** Determine the pressure gradient along this streamline. **(b)** If the upstream pressure is p_0, integrate the pressure gradient to obtain the pressure $p(x)$ for $-\infty \le x \le -a$. **(c)** Show from the result of part **(b)** that the pressure at the stagnation point ($x = -a$) is $p_0 + \rho V_0^2/2$, as expected from the Bernoulli equation.

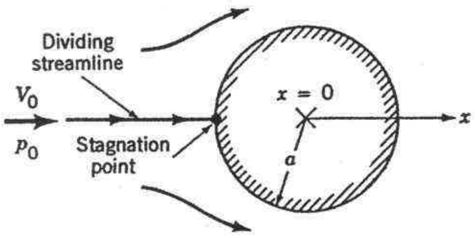

FIGURE P3.8

(a) $\frac{\partial p}{\partial s} = -\gamma \sin\theta - \rho V \frac{\partial V}{\partial s}$ but $\theta = 0$ and $\frac{\partial V}{\partial s} = \frac{\partial V}{\partial x}\frac{\partial x}{\partial s} = \frac{\partial V}{\partial x}$

$= V_0[-a^2]\left(\frac{-2}{x^3}\right) = \frac{2a^2 V_0}{x^3}$

Thus,

$$\frac{\partial p}{\partial s} = -\rho V \frac{\partial V}{\partial x} = \underline{\underline{-2\rho a^2 V_0^2\left[1 - \left(\frac{a}{x}\right)^2\right]/x^3}}$$

(b) $\int_{p_0}^{p} dp = \int_{x=-\infty}^{x} \frac{dp}{dx}\,dx$ or $p - p_0 = -2\rho a^2 V_0^2 \int_{-\infty}^{x}\left[1 - \left(\frac{a}{x}\right)^2\right]\frac{dx}{x^3}$

$$= -2\rho a^2 V_0^2 \int_{-\infty}^{x}\left[x^{-3} - a^2 x^{-5}\right] dx$$

Thus,

$$p = \underline{\underline{p_0 + \rho V_0^2\left[\left(\frac{a}{x}\right)^2 - \frac{1}{2}\left(\frac{a}{x}\right)^4\right]}} \quad \text{for} \quad -\infty \le x \le -a$$

(c) For $x = -a$, from part (b):

$$p\Big|_{x=-a} = p_0 + \rho V_0^2\left[(-1)^2 - \frac{1}{2}(-1)^4\right] = \underline{\underline{p_0 + \frac{1}{2}\rho V_0^2}}$$

Note: Bernoulli equation from point (1) where $V_1 = V_0$, $p_1 = p_0$ and $z_1 = z_0$ to point (2) where $V_2 = 0$, $z_2 = z_0$ gives

$$p_1 + \frac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \gamma z_2$$

or

$$p_2 = \underline{\underline{p_0 + \frac{1}{2}\rho V_0^2}}$$