4.33

**4.33** As a valve is opened, water flows through the diffuser shown in Fig. P4.31 at an increasing flowrate so that the velocity along the centerline is given by $\mathbf{V} = u\hat{\mathbf{i}} = V_0(1 - e^{-ct})(1 - x/\ell)\hat{\mathbf{i}}$, where $u_0$, $c$, and $\ell$ are constants. Determine the acceleration as a function of $x$ and $t$. If $V_0 = 10$ ft/s and $\ell = 5$ ft, what value of $c$ (other than $c = 0$) is needed to make the acceleration zero for any $x$ at $t = 1$ s? Explain how the acceleration can be zero if the flowrate is increasing with time.

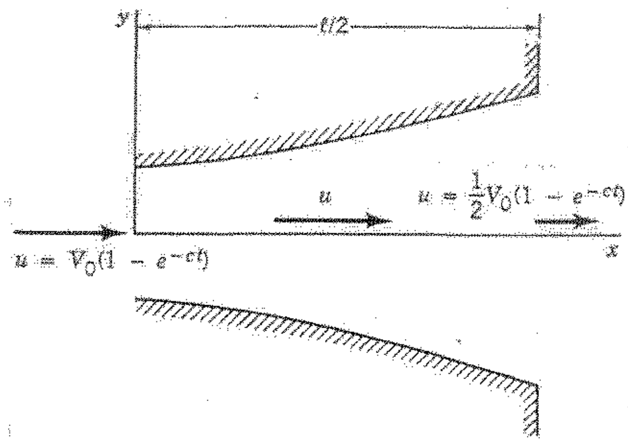


**FIGURE P4.31**

$\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V}$ With $u = u(x,t)$, $v = 0$, and $w = 0$

this becomes

$\vec{a} = \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right)\hat{\imath} = a_x\hat{\imath}$ , where $u = V_0(1 - e^{-ct})(1 - \frac{x}{\ell})$

Thus,

$$a_x = V_0(1 - \tfrac{x}{\ell})\,c\,e^{-ct} + V_0^2(1 - e^{-ct})^2(1 - \tfrac{x}{\ell})(-\tfrac{1}{\ell})$$

or

$$\underline{\underline{a_x = V_0(1 - \tfrac{x}{\ell})\left[c\,e^{-ct} - \tfrac{V_0}{\ell}(1 - e^{-ct})^2\right]}}$$

If $a_x = 0$ for any $x$ at $t = 1$ s we must have

$\left[c\,e^{-ct} - \frac{V_0}{\ell}(1 - e^{-ct})^2\right] = 0$ With $V_0 = 10$ and $\ell = 5$

$c\,e^{-c} - \frac{10}{5}(1 - e^{-c})^2 = 0$ The solution (root) of this equation is $\underline{\underline{c = 0.490\ \tfrac{1}{s}}}$

For the above conditions the local acceleration ($\frac{\partial u}{\partial t} > 0$) is precisely balanced by the convective deceleration ($u\frac{\partial u}{\partial x} < 0$). The flowrate increases with time, but the fluid flows to an area of lower velocity.