

4.68

4.68 A layer of oil flows down a vertical plate as shown in Fig. P4.68 with a velocity of $V = (V_0/h^2)(2hx - x^2)\mathbf{j}$ where V_0 and h are constants. (a) Show that the fluid sticks to the plate and that the shear stress at the edge of the layer ($x = h$) is zero. (b) Determine the flowrate across surface AB. Assume the width of the plate is b . (Note: The velocity profile for laminar flow in a pipe has a similar shape. See Video V6.13)

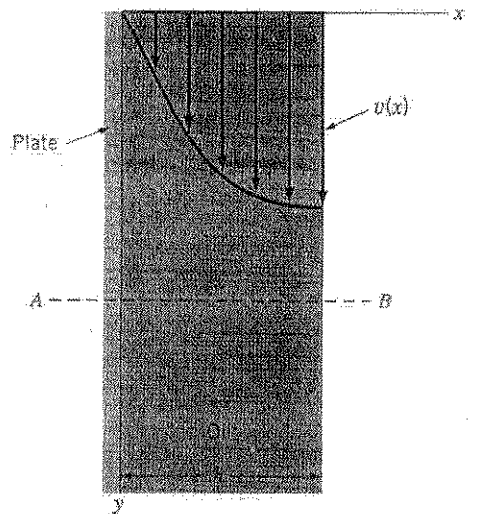


FIGURE P4.68

$$a) \quad v = \frac{V_0}{h^2} (2hx - x^2)$$

Thus,

$$v \Big|_{x=0} = \frac{V_0}{h^2} (0 - 0) = 0 \quad \text{and}$$

$$\tau \Big|_{x=h} = \mu \frac{dv}{dx} \Big|_{x=h} = \mu \frac{V_0}{h^2} [2h - 2x] \Big|_{x=h} = 0$$

Hence, the fluid sticks to the plate and there is no shear stress at the free surface.

$$b) \quad Q_{AB} = \int_{x=0}^{x=h} v \, dA = \int_{x=0}^{x=h} v \, b \, dx = \int_0^h \frac{V_0}{h^2} (2hx - x^2) b \, dx$$

or

$$Q_{AB} = \frac{V_0 b}{h^2} \left[hx^2 - \frac{1}{3} x^3 \right]_0^h = \underline{\underline{\frac{2}{3} V_0 h b}}$$