4.70

PROBLEM 5.117

GIVEN Fig. P.5.117

FIND Volume flow rate of water and minimum power input to pump.

SOLUTION The minimum pump input power occurs for frictionless flow. Assume constant fluid density. Apply Bernoulli's equation from the free water surface (0) to the pump inlet pipe at location 1 (8 in. above water surface).

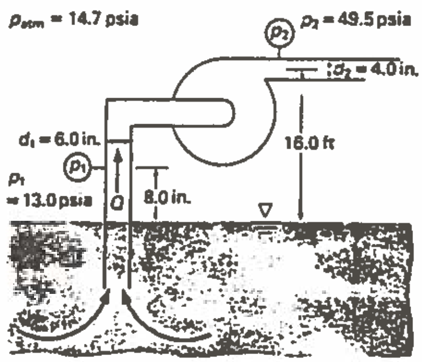

Fig. P.5.117

$$\frac{p_0}{\rho} + \frac{V_0^2}{2} + g z_0 = \frac{p_1}{\rho} + \frac{V_1^2}{2} + g z_1.$$

Now $V_0 = 0$ so

$$V_1 = \sqrt{2\left[\frac{p_0 - p_1}{\rho} + g(z_0 - z_1)\right]}.$$

Assuming 60°F water

$$V_1 = \sqrt{2\left[\frac{(14.7-13.0)\frac{lb}{in^2}\left(144\frac{in^2}{ft^2}\right)}{\left(1.938\frac{slugs}{ft^3}\right)\left(\frac{lb \cdot sec^2}{ft \cdot slug}\right)} + \left(32.2\frac{ft}{sec^2}\right)\left(-\frac{8}{12}ft\right)\right]}$$

$$= 14.5 \text{ ft/sec}$$

and

$$Q = A_1 V_1 = \frac{\pi d_1^2 V_1}{4} = \frac{\pi\left(\frac{6}{12}ft\right)^2\left(14.5\frac{ft}{sec}\right)}{4} = \boxed{Q = 2.85\frac{ft^3}{sec}}$$

The pump input power is found by writing the mechanical energy equation from the free water surface (0) to point 2 at the pump discharge.

$$\frac{p_0}{\rho} + \frac{V_0^2}{2} + g z_0 + w_s = \frac{p_2}{\rho} + \frac{V_2^2}{2} + g z_2 + \text{loss.}$$

Now $V_0 = 0$ and the minimum pump input power occurs if loss $= 0$. This gives

$$w_s = \left[\frac{p_2 - p_0}{\rho} + \frac{V_2^2}{2} + g(z_2 - z_0)\right].$$

The continuity equation gives

$$V_2 = V_1\left(\frac{A_1}{A_2}\right) = V_1\left(\frac{d_1}{d_2}\right)^2 = \left(14.5\,\frac{ft}{sec}\right)\left(\frac{6.0\,in.}{4.0\,in.}\right)^2 = 32.6\,\frac{ft}{sec}.$$

Then

$$w_s = \left[\frac{(49.5 - 14.7)\frac{lb}{in^2}\left(144\,\frac{in^2}{ft^2}\right)}{\left(1.938\,\frac{slugs}{ft^3}\right)} + \frac{\left(32.6\,\frac{ft}{sec}\right)^2}{2}\left(\frac{lb \cdot sec^2}{ft \cdot slug}\right) + \left(32.2\,\frac{ft}{sec^2}\right)(16.0\,ft)\left(\frac{lb \cdot sec^2}{ft \cdot slug}\right)\right]$$

$$= 3630\ ft \cdot lb/slug.$$

Then

$$\dot{W}_s = \dot{m}\,w_s = \rho Q w_s = \frac{\left(1.938\,\frac{slugs}{ft^3}\right)\left(2.85\,\frac{ft^3}{sec}\right)\left(3630\,\frac{ft \cdot lb}{slug}\right)}{\left(550\,\frac{ft \cdot lb}{sec \cdot hp}\right)}$$

$$\boxed{\dot{W}_s = 36.5\ hp.}$$