5.3

5.3 Water flows steadily through the horizontal piping system shown in Fig. P5.3. The velocity is uniform at section (1), the mass flowrate is 10 slugs/s at section (2), and the velocity is nonuniform at section (3). **(a)** Determine the value of the quantity $\frac{D}{Dt}\int_{\text{sys}} \rho\, d\mathcal{V}$, where the system is the water contained in the pipe bounded by sections (1), (2), and (3). **(b)** Determine the mean velocity at section (2). **(c)** Determine, if possible, the value of the integral $\int_{(3)} \rho \mathbf{V}\cdot\hat{\mathbf{n}}\, dA$ over section (3). If it is not possible, explain what additional information is needed to do so.

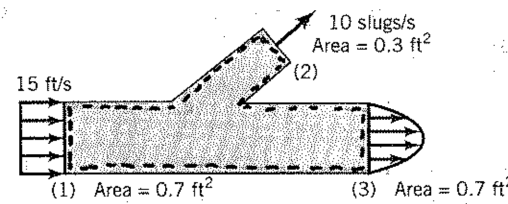

FIGURE P5.3

Use the control volume shown with the dashed lines in the figure above.

(a) From the conservation of mass principle we get
$\frac{D}{Dt}\int_{sys}\rho\, d\mathcal{V} = 0$ since $\int_{sys}\rho\, d\mathcal{V}$ is the unchanging mass of the system.

(b) $\dot{m}_2 = \rho A_2 \overline{V}_2$ thus

$$\overline{V}_2 = \frac{\dot{m}_2}{\rho A_2} = \frac{10\ \frac{slugs}{s}}{\left(1.94\ \frac{slugs}{ft^3}\right)(0.3\ ft^2)} = \underline{\underline{17.2}}\ \frac{ft}{s}$$

(c) $\dot{m}_3 = \int_{A_3} \rho \vec{V}\cdot\hat{n}\, dA$ and from the conservation of mass principle we get

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3$$

Thus

$$\dot{m}_3 = \dot{m}_1 - \dot{m}_2 = \rho A_1 \vec{V}_1 - \dot{m}_2 = \left(1.94\ \frac{slugs}{ft^3}\right)(0.7\, ft^2)\left(15\ \frac{ft}{s}\right) - 10\ \frac{slugs}{s}$$

$$\dot{m}_3 = \underline{\underline{10.4}}\ \frac{slugs}{s} = \int_{A_3} \rho \vec{V}\cdot\hat{n}\, dA$$