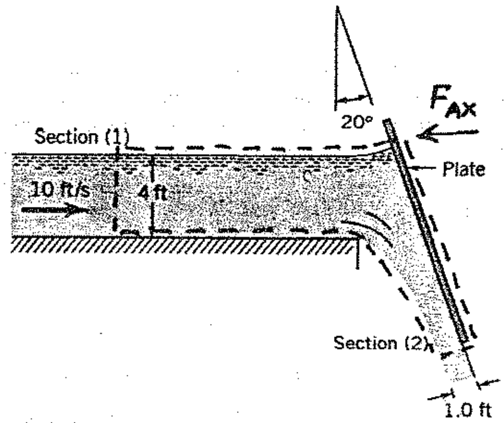
5.78

5.78 Water flows from a two-dimensional open channel and is diverted by an inclined plate as illustrated in Fig. P5.81. When the velocity at section (1) is 10 ft/s, what horizontal force (per unit width) is required to hold the plate in position? At section (1) the pressure distribution is hydrostatic, and the fluid acts as a free jet at section (2). Neglect friction.



■ FIGURE P5.81

A control volume that contains most of the plate and the water being turned by the plate as shown in the sketch above is used. Application of the horizontal x-direction component of the linear momentum equation yields

$$-V_1 \rho V_1 A_1 + V_2 \sin 20° \rho V_2 A_2 = -F_{Ax} + \frac{1}{2}\gamma_w h_1 A_1 \qquad (1)$$

From conservation of mass we obtain

$$V_2 = \frac{A_1}{A_2} V_1 = \frac{h_1}{h_2} V_1$$

Thus, Eq. 1 becomes for unit width

$$-V_1^2 \rho h_1 + \left(\frac{h_1}{h_2} V_1\right)^2 \sin 20° \rho h_2 = -F_{Ax} + \frac{1}{2}\gamma_w h_1^2$$

or

$$F_{Ax} = \frac{1}{2}\gamma_w h_1^2 + V_1^2 \rho h_1 - \left(\frac{h_1}{h_2} V_1\right)^2 \sin 20° \rho h_2$$

Then

$$F_{Ax} = \frac{1}{2}\left(62.4\ \frac{lb}{ft^3}\right)(4\ ft)^2 + \left(10\ \frac{ft}{s}\right)^2\left(1.94\ \frac{slugs}{ft^3}\right)\left(1\ \frac{lb \cdot s^2}{slug \cdot ft}\right)(4\ ft)$$

$$- \left[\left(\frac{4\ ft}{1\ ft}\right)\left(10\ \frac{ft}{s}\right)\right]^2 \sin 20° \left(1.94\ \frac{slugs}{ft^3}\right)\left(1\ \frac{lb \cdot s^2}{slug \cdot ft}\right)(1\ ft)$$

and

$$F_{Ax} = \underline{\underline{213\ lb}}$$