

6.12

6.12 The velocity components of an incompressible, two-dimensional velocity field are given by the equations

$$u = y^2 - x(1 + x)$$

$$v = y(2x + 1)$$

Show that the flow is irrotational and satisfies conservation of mass.

If the two-dimensional flow is irrotational,

$$\omega_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) = 0$$

For the velocity distribution given,

$$\frac{\partial v}{\partial x} = 2y \quad \frac{\partial u}{\partial y} = 2y$$

Thus,

$$\omega_z = \frac{1}{2} (2y - 2y) = 0$$

and the flow is irrotational.

To satisfy conservation of mass,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Since,

$$\frac{\partial u}{\partial x} = -1 - 2x \quad \frac{\partial v}{\partial y} = 2x + 1$$

then

$$-1 - 2x + 2x + 1 = 0$$

and

conservation of mass is satisfied.