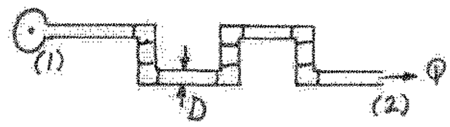
8.101

8.99 A certain process requires 2.3 cfs of water to be delivered at a pressure of 30 psi. This water comes from a large-diameter supply main in which the pressure remains at 60 psi. If the galvanized iron pipe connecting the two locations is 200 ft long and contains six threaded 90° elbows, determine the pipe diameter. Elevation differences are negligible.



$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+\left(f\frac{\ell}{D}+\Sigma K_L\right)\frac{V^2}{2g}\text{, where } p_2=30\text{ psi},\ p_1=60\text{ psi},$$

$$z_1=z_2,\ V_1=0,\ V_2=V=\frac{Q}{A}=\frac{2.3\,\frac{ft^3}{s}}{\frac{\pi}{4}D^2}=\frac{2.93}{D^2}\,\frac{ft}{s}\text{, with } D\sim ft$$

Thus,

$$p_1-p_2=\left(f\frac{\ell}{D}+\Sigma K_L\right)\frac{1}{2}\rho V^2$$

or

$$(60-30)\frac{lb}{in^2}\left(144\frac{in^2}{ft^2}\right)=\left(1+f\left(\frac{200\,ft}{D}\right)+6(1.5)+0.5\right)\left(\frac{2.93}{D^2}\frac{ft}{s}\right)^2\left(\frac{1}{2}\right)\left(1.94\frac{slugs}{ft^3}\right)$$

where we have used

$$\Sigma K_L=6K_{L_{elbow}}+K_{L_{entrance}}=6(1.5)+0.5$$

Thus,

$$49.4=\left(1+\frac{19.0f}{D}\right)\frac{1}{D^4} \qquad (1)$$

Also,

$$Re=\frac{VD}{\nu}=\frac{\left(\frac{2.93}{D^2}\right)D}{\nu}=\frac{2.93\,\frac{ft^2}{s}}{1.21\times10^{-5}\frac{ft^2}{s}\,D}\quad\text{or}\quad Re=2.42\times10^5\,\frac{1}{D} \qquad (2)$$

and from Table 8.1

$$\frac{\varepsilon}{D}=\frac{0.0005\,ft}{D} \qquad (3)$$

Finally, from Fig. 8.20:

(4)

Trial and error solution of Eqs. (1), (2), (3), and (4) for f, D, $\frac{\varepsilon}{D}$, and Re.

Normally it is easiest to guess a value of f, calculate D, etc. In this case (because of minor losse), Eq. (1) is not easy to use in this fashion. Thus, assume D, calculate f (Eq. (1)), Re (Eq. (2)), and $\frac{\varepsilon}{D}$ (Eq. (3)). Look up f in Fig. 8.20 (Eq. (4)) and compare with that from Eq. (1).

Assume $D=0.4\,ft$. Thus, $f=0.00557$, $Re=6.05\times10^5$, $\frac{\varepsilon}{D}=0.00125$
or from Fig. 8.20 $f=0.021\neq0.00557$

Assume $D=0.5\,ft$; $f=0.0551$, $Re=4.84\times10^5$, $\frac{\varepsilon}{D}=0.001$ or $f=0.0203\neq0.0551$

Assume $D=0.45\,ft$; $f=0.0243$, $Re=5.38\times10^5$, $\frac{\varepsilon}{D}=0.00111$ or $f=0.0205\neq0.0243$

Assume $D=0.44\,ft$; $f=0.0197$, $Re=5.50\times10^5$, $\frac{\varepsilon}{D}=0.00114$ or $f=0.0205\neq0.0197$

After enough trials obtain $D=\underline{\underline{0.442\,ft}}$

Note: If Fig. 8.20 (Eq. (4)) is replaced by the Colebrook equation,

(con't)

8.109 (con't)

this problem can be solved as follows.

Thus, from Eq. (1),

$f = (49.4D^5 - D)/19$ so that with the Colebrook equation (Eq. 8.35), when combined with Eqs. (2) and (3), gives

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$$

or

$$\left[\frac{19}{(49.4D^5-D)}\right]^{1/2} = -2.0 \log\left[\frac{0.0005}{3.7\,D} + \frac{2.51\,D\,\sqrt{19}}{2.42\times10^5\,(49.4\,D^5-D)^{1/2}}\right] \qquad (5)$$

Using a computer root-finding routine gives the solution to Eq. (5) as $D = 0.442$ ft which is the same as that obtained by the trial and error method above.

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