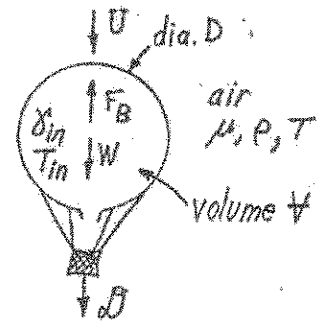
9.58

9.58 A hot air balloon roughly spherical in shape has a volume of 70,000 ft³ and a weight of 500 lb (including passengers, basket, balloon fabric, etc.). If the outside air temperature is 80 °F and the temperature within the balloon is 165 °F, estimate the rate at which it will rise under steady state conditions if the atmospheric pressure is 14.7 psi.



For steady rise $\sum F_z = 0$, or $F_B = W + \mathcal{D}$
where

$$\mathcal{D} = \text{drag} = C_D \tfrac{1}{2}\rho U^2 \tfrac{\pi}{4} D^2$$

$$F_B = \text{buoyant force} = \gamma \forall$$

and

$$W = \text{total weight} = 500 \text{ lb} + \gamma_{in} \forall$$

Now

$$\rho = \frac{p}{RT} = \frac{(14.7 \frac{lb}{in^2})(12 \frac{in}{ft})^2}{(1715 \frac{ft\cdot lb}{slug\cdot °R})(460+80)°R} = 0.00229 \frac{slugs}{ft^3}$$

$$\gamma = \rho g = (0.00229 \tfrac{slugs}{ft^3})(32.2 \tfrac{ft}{s^2}) = 0.0736 \tfrac{lb}{ft^3}$$

and

$$\gamma_{in} = \frac{p g}{R\, T_{in}} = \frac{(14.7 \frac{lb}{in^2})(12 \frac{in}{ft})^2 (32.2 \frac{ft}{s^2})}{(1715 \frac{ft\cdot lb}{slug\cdot °R})(460+165)°R} = 0.0636 \frac{lb}{ft^3}$$

Note: Since the balloon is open at the bottom, the pressure within the balloon is nearly the same as it is outside.

Thus, with $\forall = 7\times10^4 ft^3 = \frac{4\pi}{3}\left(\frac{D}{2}\right)^3$ or $D = 51.1$ ft we obtain

$$\mathcal{D} = C_D \tfrac{1}{2}(0.00229) U^2 \tfrac{\pi}{4}(51.1)^2$$

$$= 2.36\, C_D U^2 \text{ lb}, \text{ where } U \sim \tfrac{ft}{s}$$

Also,

$$W = 500 \text{ lb} + (0.0636 \tfrac{lb}{ft^3})(70{,}000 \text{ ft}^3) = 4952 \text{ lb}$$

and

$$F_B = (0.0736 \tfrac{lb}{ft^3})(70{,}000 \text{ ft}^3) = 5152 \text{ lb}$$

Thus, $F_B = W + \mathcal{D}$ gives

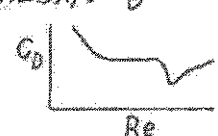
$$5152 \text{ lb} = 4952 \text{ lb} + 2.36\, C_D U^2 \quad \text{or} \quad C_D U^2 = 84.7 \qquad (1)$$

Also, $Re = \frac{UD}{\nu}$

or

$$Re = \frac{51.1 \text{ ft } U}{1.57\times10^{-4} \frac{ft^2}{s}} = 3.25\times10^5\, U \qquad (2)$$

and from Fig. 9.23 (3)



Trial and error solution: Assume C_D; obtain U from Eq.(1), Re from Eq.(2); check C_D from Eq.(3), the graph.

Assume $C_D = 0.5 \rightarrow U = 13.0 \frac{ft}{s} \rightarrow Re = 4.23\times10^6 \rightarrow C_D = 0.24 \neq 0.5$

Assume $C_D = 0.24 \rightarrow U = 18.8 \frac{ft}{s} \rightarrow Re = 6.11\times10^6 \rightarrow C_D = 0.30 \neq 0.24$

Assume $C_D = 0.30 \rightarrow \underline{U = 16.8 \frac{ft}{s}} \rightarrow Re = 5.46\times10^6 \rightarrow C_D = 0.30$ (checks)