V&V Procedures for CFD Labs

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The purpose of this document is to summarize and demonstrate V&V procedures to be used in CFD Lab reports.

Nomenclature:

S_{g1}: solution from fine grid

S_{g2}: solution from medium grid

S_{g3}: solution from coarse grid

R_g: grid convergence ratio

r_g: grid refinement ratio

P_g: order of accuracy for grid

P_{gest}: theoretical order of accuracy, 2 for 2nd order and 1 for 1st order schemes

P: ratio of accuracy of grid and theoretical accuracy

δ^*_{REg1}: grid error from Richardson Extrapolation based on fine mesh solution

U_g: grid uncertainty based on FS method

Formulae:

$\varepsilon_{g21} = S_{g2} - S_{g1}$

$\varepsilon_{g32} = S_{g3} - S_{g2}$

$R_g = \frac{\varepsilon_{g21}}{\varepsilon_{g32}}$

If monotonically converged ($0<R_g<1$), then:

$$P_g = \frac{\ln\left(\frac{\varepsilon_{g32}}{\varepsilon_{g21}}\right)}{\ln\left(r_g\right)}$$

$$P = \frac{P_g}{P_{gest}}$$

$$\delta^*_{REg1} = \frac{\varepsilon_{g21}}{\left(r_g^{P_g} - 1\right)}$$

$$U_g = \begin{cases} \left(2.45 - 0.85P\right)\left|\delta^*_{REg1}\right| & \text{if} \quad 0 < P \le 1 \\ \left(16.4P - 14.8\right)\left|\delta^*_{REg1}\right| & \text{if} \quad P > 1 \end{cases}$$

Following examples demonstrate grid studies for friction factor of laminar pipe flows.

Example:

$$r_g = \sqrt{2}$$

$$\varepsilon_{g21} = S_{g2} - S_{g1} = -0.0027916$$

$$\varepsilon_{g32} = S_{g3} - S_{g2} = -0.0157815$$

$$P_g = \frac{\ln(\frac{\varepsilon_{g32}}{\varepsilon_{g21}})}{\ln(r_g)} = 2.49907$$

$$P = \frac{P_g}{P_{gest}} = \frac{2.49907}{2} = 1.249535$$

$$\delta^*_{REg1} = \frac{\varepsilon_{g21}}{(r_g{}^{P_g} - 1)} = -0.0006$$

$$P > 1$$

$$U_g = (16.4P - 14.8) * \left|\delta^*_{REg1}\right| = 0.003415424$$