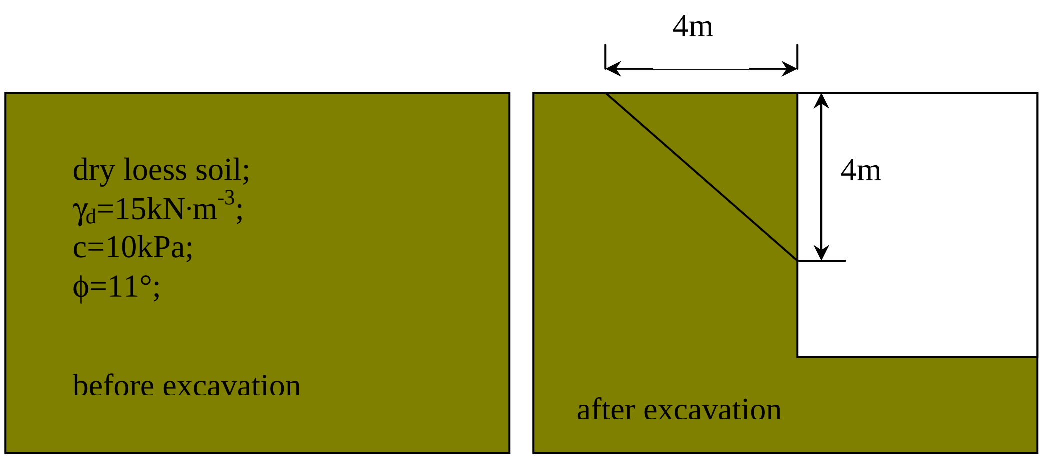
**The University of Iowa**
**Dept. of Civil & Environmental Engineering**
**53:030 SOIL MECHANICS**
**Final Examination**
**Fall Semester 2005**
**200 points**

**<u>Question #1</u>: (40 points)**

A 6m vertical cut has been excavated from a dry cohesive loess soil deposit. For the plane shown passing through the vertical cut face, estimate whether or not a shear failure is likely. Show all of your work to receive credit.

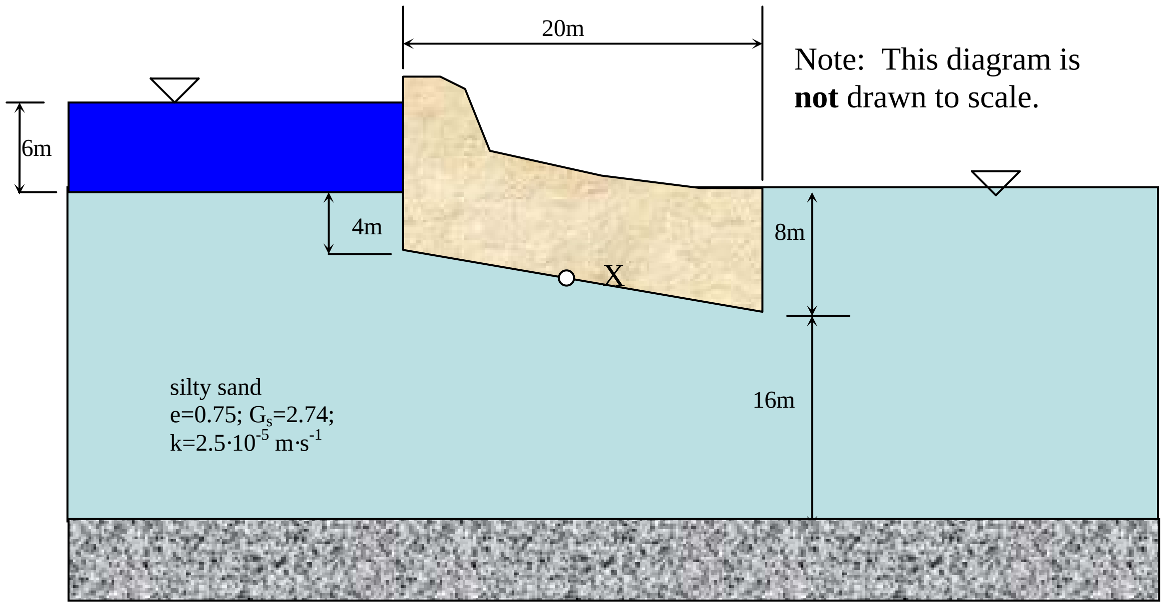
a. Estimate the average normal stress and shear stress along the plane;
b. Based on these values, determine whether or not shear failure is likely.




**<u>Question #2</u>: (40 points)**

Consider the reservoir, a portion of which is shown in the diagram below.

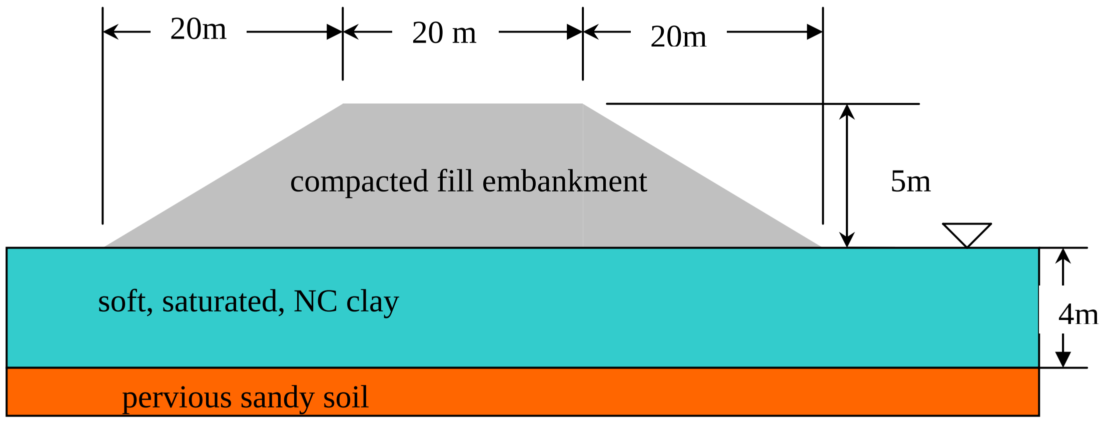
a. In your exam booklet, draw the flow domain to scale and then draw a good, neat flownet that satisfies all of the necessary requirements.
b. Based on your flownet and the soil properties, what is the rate of fluid seepage out of the reservoir, per unit length in the out-of-plane direction?
c. Based on your flownet, compute the fluid pressure at a point X midway along the base of the retention structure.
d. Based on your flownet, what is the factor of safety against liquefaction in the critical region of the flow domain?
e. What depth of water in the reservoir would be necessary to cause liquefaction in the critical region?




**<u>Question #3</u>: (40 points)**

To raise the ground level for a roadway surface, a one-kilometer long embankment whose cross-section is shown below will be constructed out of compacted fill. The relevant construction codes require that the fill soil ($G_s$=2.68) be compacted to a minimum dry unit weight of 18 $kN \cdot m^{-3}$. In the borrow pit, the loose soil has e=0.85, and w=0.10.

a. How many 100 ton truckloads of moist soil will need to be brought to the embankment construction site from the borrow site?
b. If the target moisture content at which the soil will be compacted is w=0.13, how much water will have to be added to each 100 ton load of the fill soil?
c. At w=0.13, what will be the estimated degree of saturation of the compacted fill?



**<u>Question #4</u>: (40 points)**

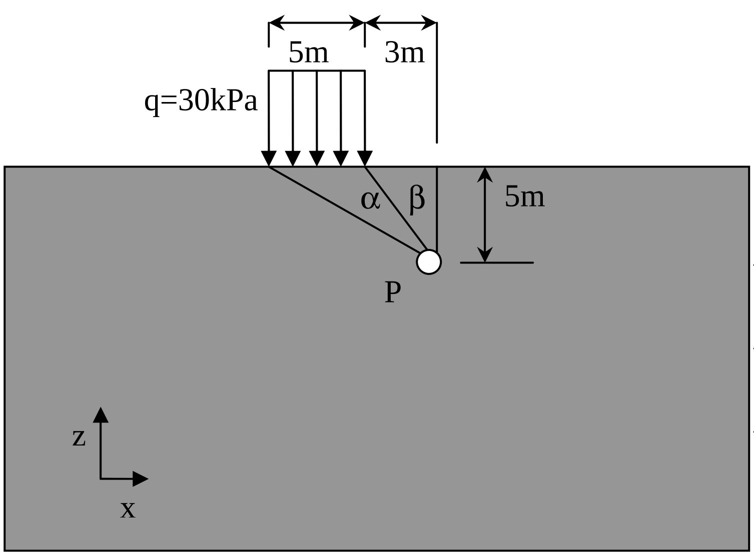
Consider the embankment of Question #3 built on a soft, saturated, normally consolidated clay soil. The clay has the following properties: $C_s$=0.02; $C_c$=0.48; $e_0$=0.90; $G_s$=2.60; $c_v=1.6\text{m}^2\cdot\text{yr}^{-1}$

a. Estimate the average stress increase in the clay layer beneath the center of the embankment.
b. Calculate the ultimate consolidation settlements that will occur beneath the center of the embankment.
c. How long will it take for 90% of this consolidation settlement to occur? (Use one dimensional consolidation theory to answer this question.) The nondimensional time constant for 90% consolidation is $T_{90}$ = 0.848.

**<u>Question #5</u>: (40 points)**

The figure below shows a homogeneous, sandy soil deposit with a horizontal ground surface. Before the strip load is applied, the stresses at point P are as follows: vertical stress $\sigma_v$=90 kPa; horizontal stress $\sigma_h$=45kPa.

a. Compute the maximum shear stress at point P **before** the surface pressure is applied.
b. Using the information provided in the figure below, compute the major and minor principal stresses at point P **after** the uniform strip load is applied.
c. What are the respective orientations of the principal planes at point P after the surface pressure is applied?




$$\Delta\sigma_{zz} = \frac{q}{\pi}[\alpha + \sin(\alpha)\cos(\alpha + 2\beta)]$$

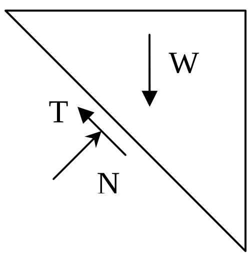
$$\Delta\sigma_{xx} = \frac{q}{\pi}[\alpha - \sin(\alpha)\cos(\alpha + 2\beta)]$$

$$\Delta\tau_{xz} = \frac{q}{\pi}[\sin(\alpha)\sin(\alpha + 2\beta)]$$

**The University of Iowa**
**Dept. of Civil & Environmental Engineering**
**53:030 SOIL MECHANICS**
**Final Examination Solution**
**Fall Semester 2005**
**200 points**

## Question #1 Solution:

A free-body diagram of the potentially unstable portion of the cut is shown below:




W=γ*V=15kN·m$^{-3}$*8m$^3$
= 120 kN
N=Wcos(45º)=84.85 kN
T=Wsin(45º)=84.85 kN

$$\sigma_n = N / A = \frac{\left(120kN * \frac{\sqrt{2}}{2}\right)}{\left(4m * \sqrt{2} * 1m\right)} = 15kPa$$

$$\tau = T / A = \frac{\left(120kN * \frac{\sqrt{2}}{2}\right)}{\left(4m * \sqrt{2} * 1m\right)} = 15kPa$$

$$\tau_{\max} = c + \sigma_n \tan(\phi) = 10kPa + 15kPa * \tan(11^\circ)$$
$$= 12.92\text{kPa}$$

Since the actual average shear stress on the plane exceeds the average shear strength of the soil on the plane, **shear failure is highly likely**.

## Question #2 Solution:

a. A proper flownet on the domain drawn to scale is shown in the figure below.

b. The flownet has $N_f$=7 and $N_d$=17. The overall flow rate beneath the structure is:

$$q = kH\frac{N_f}{N_d} = 2.5 \cdot 10^{-5} m \cdot s^{-1} * 6m * \frac{7}{17} = 6.2 \cdot 10^{-5} m^3 / s / m$$

c. Taking the mudline at the reference datum, the head at the upstream mudline is 6m, and the head at the downstream mudline is 0m. The headloss between any two adjacent equipotentials is thus:

$$\Delta h = \frac{H}{N_d} = \frac{6m}{17} = 0.353m$$

The head at point X is: $h_X = 6m - 6.5\Delta h = 6m - 6.5*(0.353m) = 4.21m$

The fluid pressure at X can be obtained from the head as follows:

$$p_X = \gamma_w\left(h_X - (h_z)_X\right) = 9.81kN \cdot m^{-3} * \left(4.21m - (-6m)\right) = 100.16kPa$$

d. In the critical region, the flow is almost straight upward, and the approximate upward hydraulic gradient in that region is as follows:

$$i_{ave} = \frac{\bar{n}_d \Delta h}{D} = \frac{\frac{1}{2}(4+7)) * 0.353m}{8m} = 0.243$$

The critical hydraulic gradient for these soil properties is:

$$i_{cr} = \frac{\gamma_b}{\gamma_w} = \frac{G_s - 1}{1+e} = \frac{1.74}{1.75} = 0.994$$

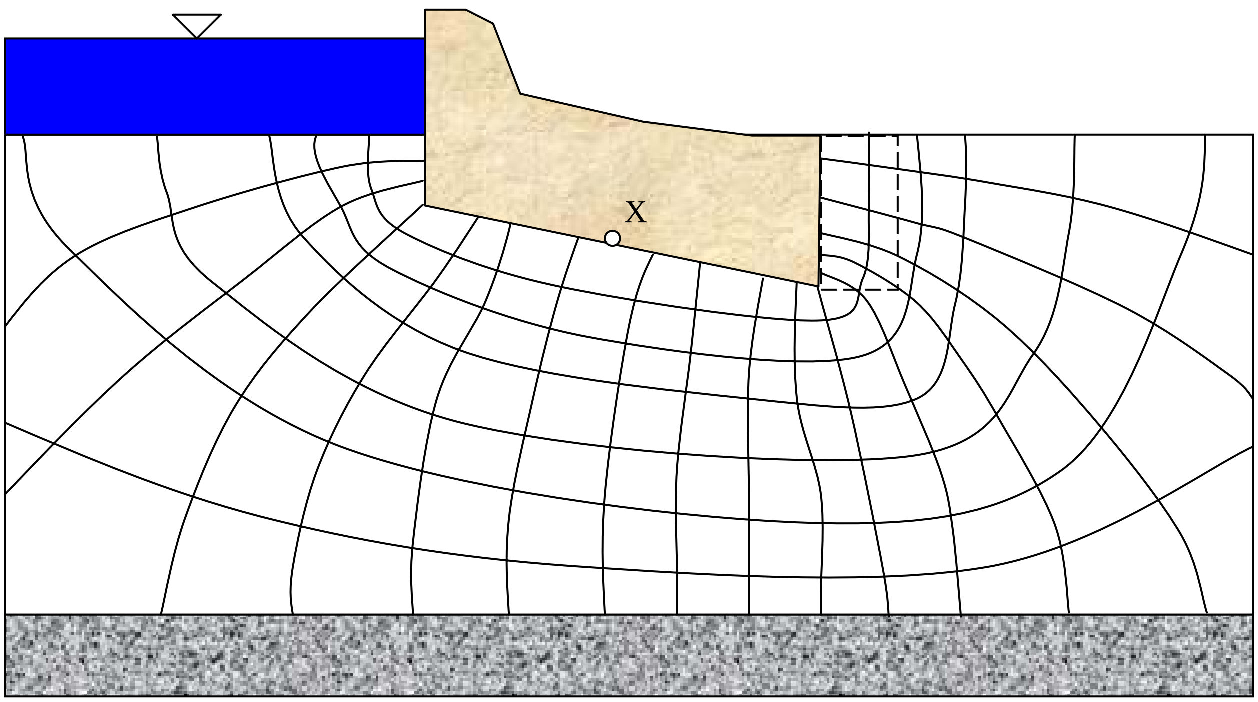
Therefore the factor of safety against liquefaction in the critical region is:

$$FS = \frac{i_{cr}}{i_{ave}} = \frac{0.994}{0.243} = 4.1$$

e. The required depth of water in the reservoir $H_{crit}$ to cause liquefaction in the critical region can be computed as follows:

$$i_{cr} = 0.994 = i_{ave} = \frac{\bar{n}_d (\Delta h)_{crit}}{D} = \frac{\bar{n}_d \frac{H_{crit}}{N_d}}{D}$$

Solving for $H_{crit}$ yields: $H_{crit} = i_{ave} * D * \frac{N_d}{\bar{n}_d} = 0.994 * 8m * \frac{17}{5.5} = 24.6m$




**<u>Question #3 Solution:</u>**

a) $V_{embankment}$=2*(20m*5m)*1000m
= 200,000 $m^3$

$W_{embankment}$= $\gamma_{embankment}$*$V_{embankment}$ = $\gamma_d$*(1+w)*200,000$m^3$
= 18kN$m^{-3}$*(1.10)*200,000$m^3$ =3.96*$10^6$kN

Note: 1 ton = 1000kg*9.81m/$s^2$ = 9.81kN, so
100 tons = 981kN

No. truckloads = 3.96*$10^6$kN /(981kN/load)
= 4,037 truckloads

b) In 981 kN of soil (1 truckload), the weight of solids is:

$W_s$=W/(1+w)=981kN/1.10=891.82kN

At w=0.10, the corresponding weight of moisture is:

$W_w$=w*Ws=0.10*891.82kN=89.182kN

If the moisture content of the soil were raised to w=0.13, then the moisture weight corresponding to 891.82kN of dry soil would be:

$W_w$=w*Ws=0.13*891.82kN=115.94kN.

The increase in water weight is: 115.94kN-89.182kN=26.75kN

**So, 26.75kN should be added to each 981kN truckload of soil to raise the moisture content to w=0.13.**

c)

In the compacted fill, the dry unit weight of the soil is 18kN/m$^3$. The corresponding void ratio of the soil is:

$$e=\frac{G_s\gamma_w}{\gamma_d}-1=\frac{2.68*9.81kN\cdot m^{-3}}{18kN\cdot m^{-3}}-1=0.4606$$

$$S=\frac{wG_s}{e}=\frac{0.13*2.68}{0.4606}=0.756=75.6\%$$

## <u>Question #4 Solution:</u>

a) The moist unit weight of the compacted fill is:

$\gamma=\gamma_d*(1+w)= 18kNm^{-3}*(1.10)=19.8\ kNm^{-3}$

The corresponding stress increase beneath the center of the embankment will be simply: $\Delta\sigma_v=\gamma h=19.8kN\cdot m^{-3}*5m=99kPa.$

b)

The consolidation settlements beneath the center of the embankment:

At the center of the clay layer, before the embankment is constructed:

$$(\sigma_v)_o'=2m(\gamma_b)_{clay}=2m*\frac{\gamma_w(G_s-1)}{1+e}=2m*\frac{9.81kN\cdot m^{-3}*(2.60-1)}{1.0+0.90}$$
$$=16.52\text{ kPa}$$

After the embankment is constructed and the clay soil fully consolidates:

$$(\sigma_v)_f'=(\sigma_v)_o'+\Delta\sigma_v=16.52\ kPa+$$
$$=16.52\text{ kPa}+99\text{ kPa}$$
$$=115.52\text{ kPa}$$

Since the soil is normally consolidated,

$$S_c = \frac{H_o}{1+e_o}\Delta e = \frac{H_o}{1+e_o}C_c \log\left(\frac{(\sigma_v)_f'}{(\sigma_v)_o'}\right) = \frac{4m}{1.90}*0.48*\log\left(\frac{115.52}{16.52}\right)$$

$$= 0.85\text{m}$$

c)

$$t_{90} = \frac{0.848*(H_{dr})^2}{c_v} = \frac{0.848*(2m)^2}{1.6m^2yr^{-1}} = 2.12yrs$$

**Question #5 Solution:**

a)

$$\sigma_1' = \sigma_v' = 90kPa$$

$$\sigma_3' = \sigma_h' = 45kPa$$

The resulting effective stress Mohr's Circle has a center at $\sigma_c = \frac{1}{2}(90+45) = 67.5kPa$, and a radius of $r = \frac{1}{2}(\sigma_1' - \sigma_3') = 22.5kPa$

$\tau_{\max} = r = 22.5kPa.$

b)

$$\beta = \tan^{-1}\left(\frac{3m}{5m}\right) = 0.5404 \text{ radians}; \ \alpha+\beta = \tan^{-1}\left(\frac{8m}{5m}\right) = 1.0122 \text{ radians}$$

$$\alpha = \alpha+\beta-\beta = 0.4718 \text{ radians}$$

$$\Delta\sigma_{zz} = \frac{30kPa}{\pi}\left[0.4718 + \sin(0.4718)\cos(0.4718 + 2*0.5404)\right] = 4.584 \text{ kPa}$$

$$\Delta\sigma_{xx} = \frac{30kPa}{\pi}\left[0.4718 - \sin(0.4718)\cos(0.4718 + 2*0.5404)\right] = 4.426 \text{ kPa}$$

$$\Delta\tau_{xz} = \frac{30kPa}{\pi}\left[\sin(0.4718)\sin(0.4718 + 2*0.5404)\right] = 4.339 \text{ kPa}$$

$$\sigma_{zz} = 90kPa + 4.584kPa = 94.58kPa$$

$$\sigma_{xx} = 45kPa + 4.426kPa = 49.43kPa$$

$$\tau_{xz} = 4.34 \text{ kPa}$$

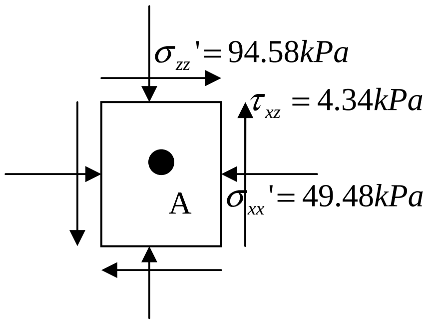
$$\sigma_c = \tfrac{1}{2}(\sigma_{zz} + \sigma_{xx}) = 72.0kPa; \qquad r = \left[\left(\frac{\sigma_{zz} - \sigma_{xx}}{2}\right)^2 + \tau_{xz}^{\ 2}\right]^{1/2} = 22.99kPa$$

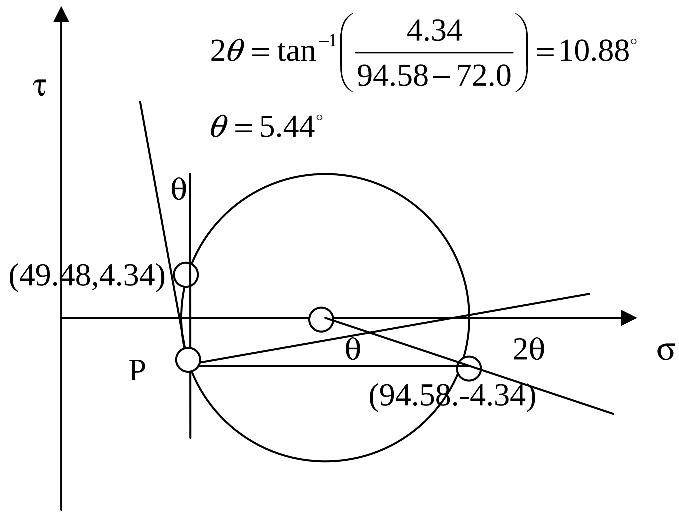
$$\sigma_1 = \sigma_c + r = 94.99kPa; \ \sigma_3 = \sigma_c - r = 49.01kPa;$$

c) Determination of the principal planes:

The strip load does not change the stresses at point X appreciably, so we can expect that the major principal plane will be close to horizontal, and the minor principal plane will be close to vertical.

Nevertheless, a precise quantification of the principal plane angles should be carried out, using the Pole Method as follows:




- the major princ. plane makes angle θ=5.44º ccw wrt horizontal;
- the minor princ. plane make angle θ =5.44º ccw wrt vertical