Period #16: Soil Compressibility and Consolidation (II)

A. Review and Motivation

(1) Review:

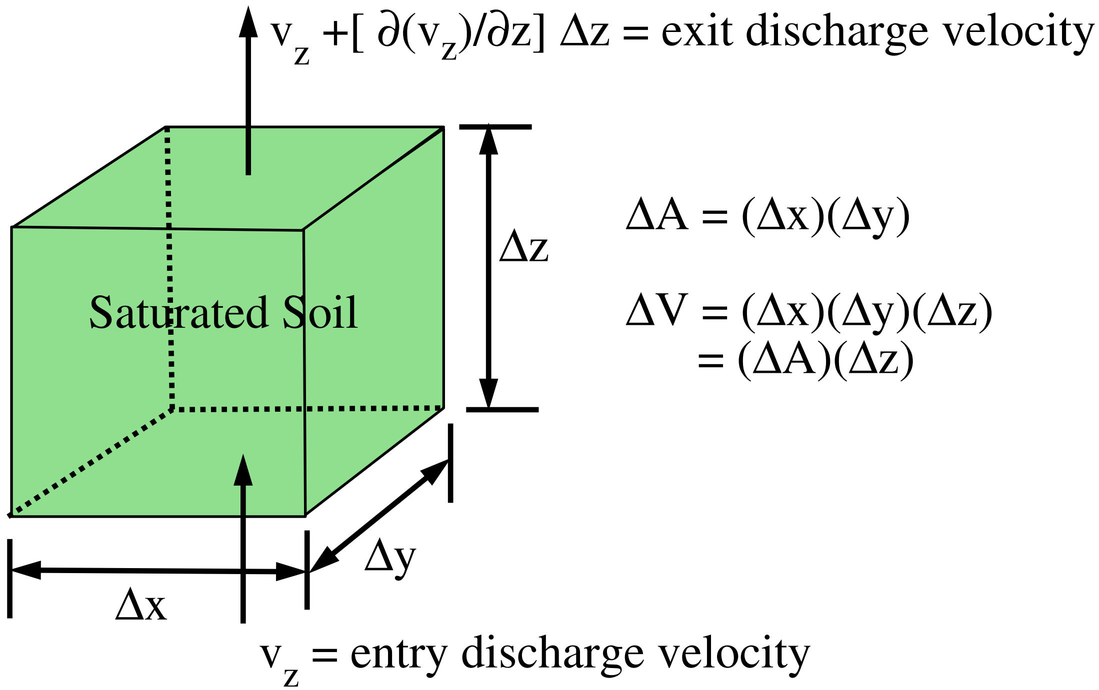
- In most soils, changes in *total* volume are associated with reductions in *void* volume. The volume change of the soil grains is negligible.
- Changes in soil volume are produced by *effective* stresses.
- When loads are applied to soils, the increased compressive stresses can initially be taken up by the pore fluid.
- With time, though, the pore fluid pressure dissipates, and the increased compressive stresses are transferred to the soil skeleton.
    - As this *gradually* occurs, the soil will compress.

(2) Motivation:

- The important question that remains to be answered, is how long can this take?
- To better understand this, a common engineering consolidation model is presented here.

B. Rate of Consolidation in One–Dimension

- To begin, consider a very small element of soil being subjected to one–dimensional consolidation in the z–direction.



$\Delta A = (\Delta x)(\Delta y)$

$\Delta V = (\Delta x)(\Delta y)(\Delta z) = (\Delta A)(\Delta z)$

- Rate of fluid mass outflow $= \rho_w \Delta A \left\{ v_z + [\partial(v_z)/\partial z]\,\Delta z \right\}$
- Rate of fluid mass inflow $= \rho_w \Delta A\, v_z$
- Net rate of fluid mass outflow = outflow – inflow

$$= \rho_w \Delta A\, [\partial(v_z)/\partial z]\,\Delta z = \rho_w \Delta V\, \partial(v_z)/\partial z$$

- fluid mass conservation for this element:

net rate of fluid mass outflow = net rate of internal fluid mass decrease

$$\rho_w \Delta V \;\partial(v_z)/\partial z = -\,\partial\{M_w\}/\partial t$$
$$= -\partial\{\rho_w e\, \Delta V_s\}/\partial t$$
$$= -\rho_w \Delta V_s\, \partial\{e\}/\partial t$$

Aside: Note these assumptions:
a) fluid is incompressible (i.e. $\partial(\rho_w)/\partial t = 0$); and
b) soil grains are also incompressible (i.e. $\partial(\Delta V_s)/\partial t = 0$).

$$\therefore \quad \rho_w \Delta V\, \partial(v_z)/\partial z = -\rho_w \Delta V_s\, \partial\{e\}/\partial t,$$

$$\partial(v_z)/\partial z = -\,(\Delta V_s/\Delta V)\, \partial\{e\}/\partial t$$

$$\partial(v_z)/\partial z = -\,(1+e_0)^{-1}\, \partial\{e\}/\partial t$$ statement of fluid mass conservation.

- For 1–D flow in the z–direction, Darcy's Law gives $v_z = -k_z\, \partial h/\partial z$,
  where:
  $h = h_z + p_w/\gamma_w$
  $k_z =$ permeability in the z–direction

∴ fluid mass conservation can be re−written as:

$$\partial(-k_z \partial h/\partial z)/\partial z = -(1+e_0)^{-1}\, \partial e/\partial t$$

$$k_z\, \partial^2 h/\partial z^2 = (1+e_0)^{-1}\, \partial e/\partial t$$

Note the assumption that k_z is constant in the z−direction.:

- This expression needs to be re−written in terms of pore pressure.

$$h = h_z + (p_w/\gamma_w) = h_z + (p_{hyd} + u)/\gamma_w$$

- Note: In the above expression, the fluid pressure p_w has been expressed as $p_w = p_{hyd} + u$, where: p_{hyd} is the equilibrium hydrostatic pore−pressure in the soil before a load was applied, and u is the excessore pore−pressure in the soil due to the application of a load.

- Since both h_z and p_{hyd} are linear functions of z, $\partial^2 h/\partial z^2 = (1/\gamma_w)\, \partial^2 u/\partial z^2$.

$$\therefore\ (k_z/\gamma_w)\, \partial^2 u/\partial z^2 = (1+e_0)^{-1} \partial e/\partial t$$

- The remaining task is to relate the void ratio e to excess pore pressure u.

•This is done in two steps:

a) Note that when a *constant* and *uniform* load is applied to a soil deposit and consolidation is occuring, the vertical stress in the soil is constant.

$$0 = \partial(\sigma_v)/\partial t = \partial(\sigma'_v)/\partial t + \partial(p_w)/\partial t$$

$$\text{Thus, } \partial(\sigma'_v)/\partial t = -\partial(p_w)/\partial t$$
$$= -\partial u/\partial t$$

That is, an *increase* in vertical effective stress σ'_v is achieved by a *reduction* of excess pore pressure u.

b) Now to relate σ'_v to the void ratio e, the e vs. σ_v' behavior of the soil is used.

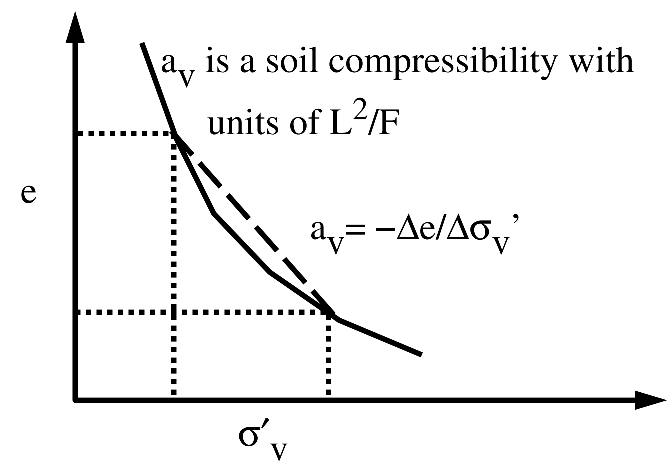
- From the figure shown, by *linearizing* the soil behavior over a given range of σ'_v and e:

$$\Delta e = -a_v \Delta \sigma'_v$$

- From the preceding relation between σ'_v and u, it follows that:

$$\Delta e = a_v \Delta u$$

- Expressing this in rate form, $\partial e/\partial t = a_v \partial u/\partial t$.



- Putting it all together, the final statement of fluid mass conservation is:

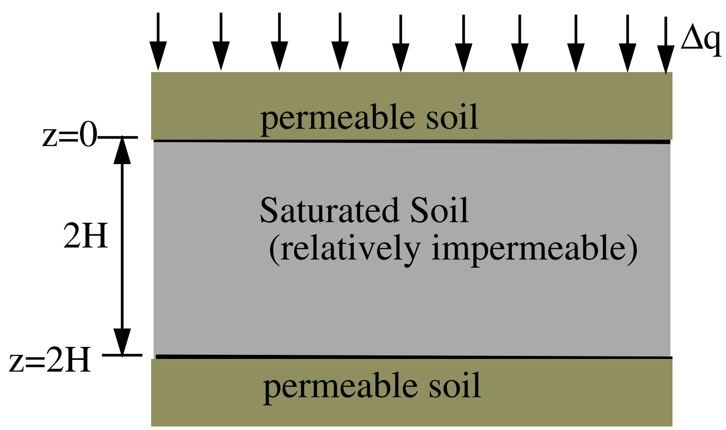
$$(k_z/\gamma_w)\, \partial^2 u/\partial z^2 = [\, a_v/(1+e_0)]\, \partial u/\partial t$$

or

$$c_v\, \partial^2 u/\partial z^2 = \partial u/\partial t : \qquad \text{One-dimensional consolidation equation}$$

where $c_v = k_z(1+e_0)/(a_v \gamma_w)$: consolidation coefficient (units are L^2/T)

C. Solutions of the Consolidation Equation



Initial conditions for excess pore–pressure in the low–permeability soil layer:

$u = u_0 = \Delta q$ for $z \in (0,2H)$ at $t=0$

Boundary conditions for excess pore–pressure in the low–permeability soil layer:

$u = 0$ at $z = 0$ for $t \in [0, \infty)$
$u = 0$ at $z = 2H$ for $t \in [0, \infty)$

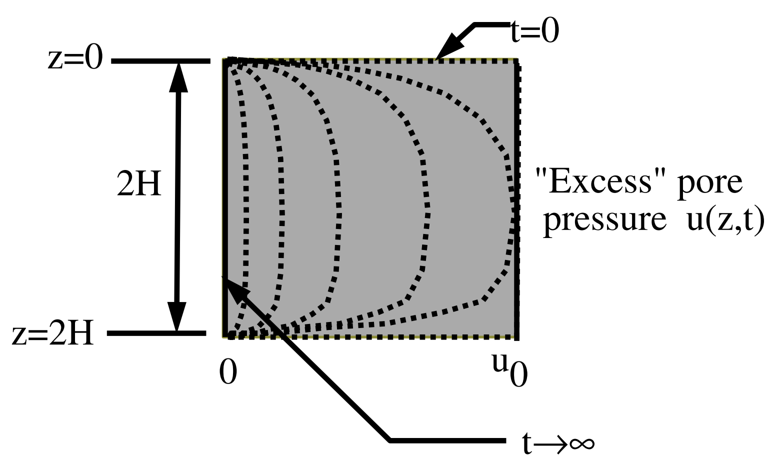
- The solution to the 1–D consolidation equation for these boundary conditions and initial conditions is:

$$u(z,t) = \sum_{m=0}^{\infty} \left\{ (2u_0/M) \sin\left(Mz/H\right) \right\} \exp\left\{-M^2 T_v\right\}$$

where: $M = (\pi/2)(2m+1)$ and

$T_v = c_v t / H^2$: non–dimensional time factor

H : maximum drainage distance in the soil layer



- The next task is to relate the excess pore–pressure u(z,t) to the actual compression of the soil layer.

- At a given time t, and a given location z in the soil layer, define the local degree of consolidation as:

$$U_z(z,t) = \left[u_0 - u(z,t)\right] / u_0 = 1 - \left[u(z,t) / u_0\right]$$

Observations:

- when $u(z,t) = u_0$, then $U_z(z,t) = 0$, which means that the soil at (z,t) has not yet begun to consolidate; and
- when $u(z,t) = 0$, then $U_z(z,t) = 1$, which means that the soil at (z,t) has fully consolidated.

- Now define the average degree of consolidation U for the whole soil layer at a given time t as:

$$U(t) = (1/2H) \int_0^{2H} U_z(z,t)\, dz = (1/2H) \int_0^{2H} [1 - u(z,t)/u_0]\, dz$$

$$= 1 - \sum_{m=0}^{\infty} (2/M^2) \exp\left\{-M^2 T_v\right\}$$

U vs. T_v is plotted in Figure 10.24 and tabulated in Table 10.5 of the textbook.

D. Applications

Example 16.1: A 3m thick double–drained layer of saturated clay soil was subjected to a surcharge loading and achieved 90% primary consolidation in 75 days. Find the coefficient of consolidation c_v.

Solution:

- The dimensionless time factor for soils is: $T_v = (c_v\ t) / H^2$.

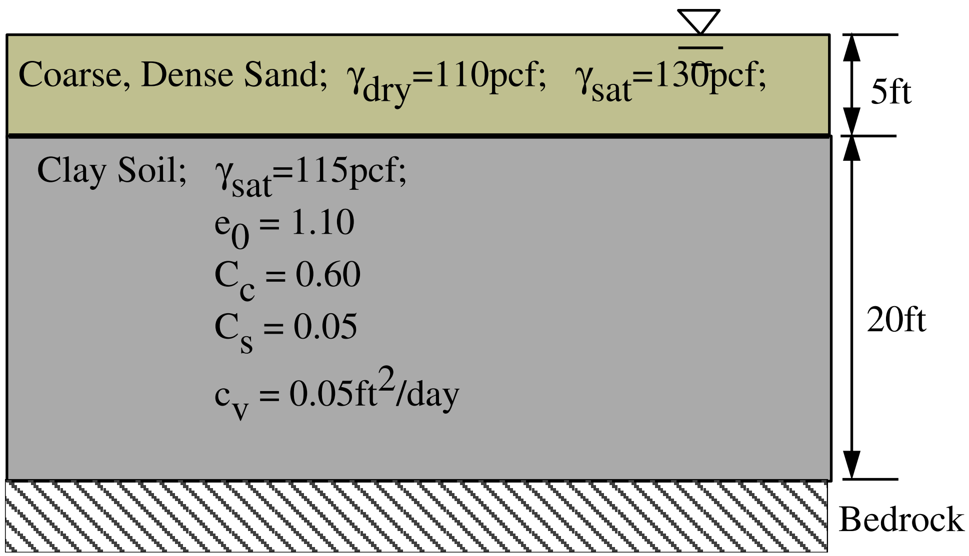
$\therefore T_{90} = (c_v\ t_{90}) / H^2 = [c_v * 75 \text{days}]/(1.5\text{m})^2$

- From the U vs. T_v plot on page 288 of the text, $T_{90} = 0.85$

- Solving for c_v gives, $\mathbf{c_v = 0.0255 m^2/day = 2.95E{-}7 m^2/sec.}$

Example 16.2: Consider the soil profile shown. The phreatic surface now coincides with the ground surface, but a long time ago it used to be at a depth of 5 feet below the found surface. Asume that a uniform pressure of 400 psf is to be applied over a large area.

a) Use the location of the phreatic surface a long time ago to compute the pre−consolidation vertical effective stress in the clay layer;
b) Estimate the ultimate settlement of the ground surface due only to primary consolidation of the clay layer; and
c) How long will it take to achieve 50% and 90% consolidation of the clay layer under the imposed loading?



Solution:

a) First, compute the vertical effective stress at the center of the clay layer before the uniform pressure is applied.

$$(\sigma'_v)_0 = 5' * (130-62.4)\text{pcf} + 10' * (115-62.4)\text{pcf}$$
$$= 864 \text{ psf}$$

A long time ago, the phreatic surface was at the clay–sand interface. At that time, the vertical effective stress was:

$$(\sigma'_v) = 5' * (110)\text{pcf} + 10' * (115-62.4)\text{pcf}$$
$$= 1076 \text{ psf}$$

Since the vertical effective stress a long time ago was greater than what the current vertical effective stress is, the clay soil is over–consolidated, and **the pre–consolidation stress $(\sigma'_v)_c$ = 1076psf**.

b) Estimate the ultimate settlement due to application of the load:

$$\Delta H = H_0 \Delta e/(1+e_0)$$

$$= (20\text{ft}/2.1)*[-C_s \log[(\sigma'_v)_c /(\sigma'_v)_0] - C_c \log[(\sigma'_v)_f /(\sigma'_v)_c]\,]$$

$$= 9.524\text{ft} * [-0.05 \log(1076/864) - 0.6 \log(1264/1076)]$$

$$= 9.524\text{ft} * [\;-.0048 \;\; - .0420]$$

ΔH =– 0.445 ft = –5.34in = ultimate settlement.

c) Solution: The dimensionless time factor for soils is: $T_v = (c_v\ t)/H^2$.

From the plot of U vs. T_v on page 292 of the text: $T_{50} = 0.18$ and $T_{90} = 0.85$

Thus, $t_{50} = (T_{50}H^2)/c_v = 0.18 * (20ft)^2/ (0.05ft^2/day)$

$= 1440$ days (or about 4 years!)

In a similar way,

$t_{90} = (T_{90}H^2)/c_v = 0.85 * (20ft)^2/ (0.05ft^2/day)$

$= 6800$ days (or about 18.6 years!)

E. Summary:

- From this second example, we see that the times over which consolidation actually occurs can be over *many years and even decades*.

- The important factors that determine how long it will take for consolidation settlements to occur are:

  1) **the consolidation coefficient c_v** which is proportional to a soil's permeability k. The smaller c_v (and the smaller k) the longer it will take for a soil to consolidate.

  2) **the maximum drainage distance H.** From the above examples, note that consolidation times are proportional to H^2.