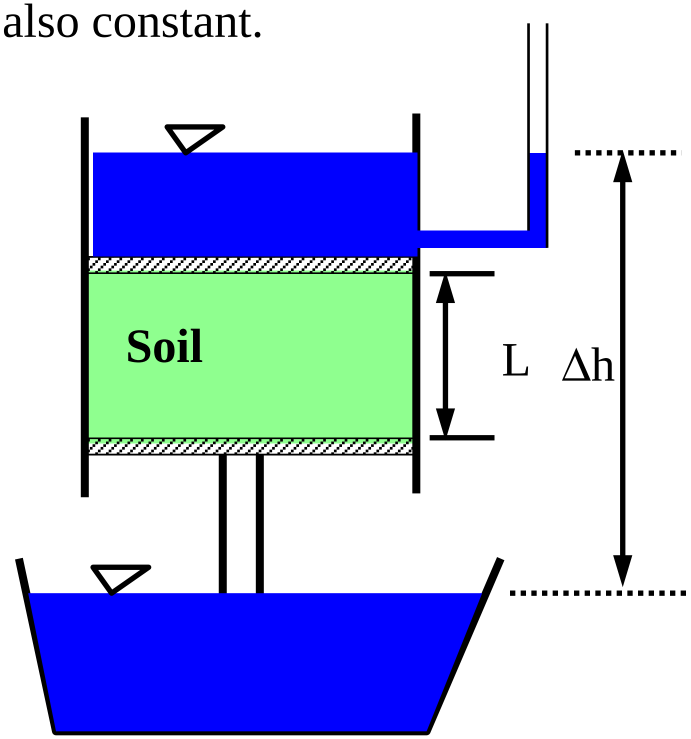
Period #8: Fluid Flow in Soils (II)

A. Measuring Permeabilities in Soils

1. The Constant Head Test (For Coarse−Grained Soils):

- Upstream and downstream head elevations are maintained at *constant* levels.
- The head difference across the soil is a constant value Δh.
- The hydraulic gradient i across the sample is also constant.
  i = hydraulic gradient in the soil (Δh/L)



$Q = vAT$
= volumetric flow through the soil over an elapsed time T.

$q = Q/T = vA$
= rate of volume flow

$v = q/A$ = the so−called discharge velocity

A = the cross−sectional area of the soil sample

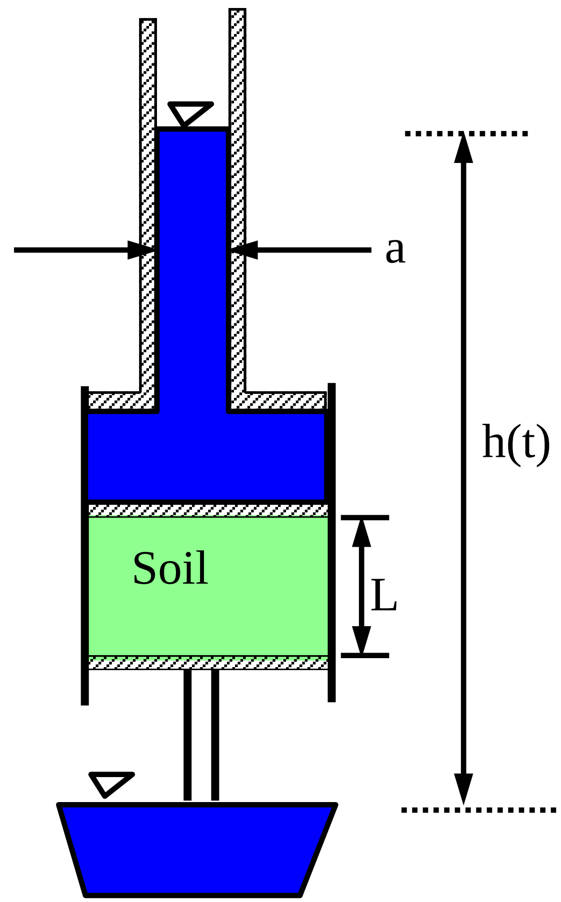
Recall from Darcy's Law that: $v = ki$

From a constant head test, soil permeability k can be computed as:

k = Q/(AiT) where

2. The Falling Head Test (relatively impermeable soils).

- The total head difference *h(t)* across the sample changes with time.
  −> The flow rate through the soil is not constant.



q_{in} = Flow rate into the soil
$= -a\, dh(t)/dt$

q_{out} = Flow rate out of the soil
$= kA * i(t)$
$= kA * h(t)/L$

Conservation of fluid mass gives:

$$q_{in} = q_{out}$$
$$-a\, dh(t)/dt = kA * h(t)/L$$

This represents a first order ODE to be solved for *h(t)*:

Solution: $\ln(h(t)) - \ln(h_o) = -kAt/(aL)$

The permeability *k* of the soil can thus be determined from this test as follows:

$$k = -\ln(h(t)/h_o)\, aL/[At]$$

3. The well−drawdown test: used to measure in−situ permeabilities of soils

Procedure:

a) drill a test well and two observation wells;

b) continuously pump water out of the test well until water levels in all three wells achieve equilibrium levels;

c) once steady state flow is achieved, the radial flow rate q(r) is constant;

$q = ki(r)A(r)$ = constant as function of r

where:

k = soil permeability

i = local gradient = dh/dr

A = cross−sectional area of flow $= 2\pi rh(r)$

Assumptions:

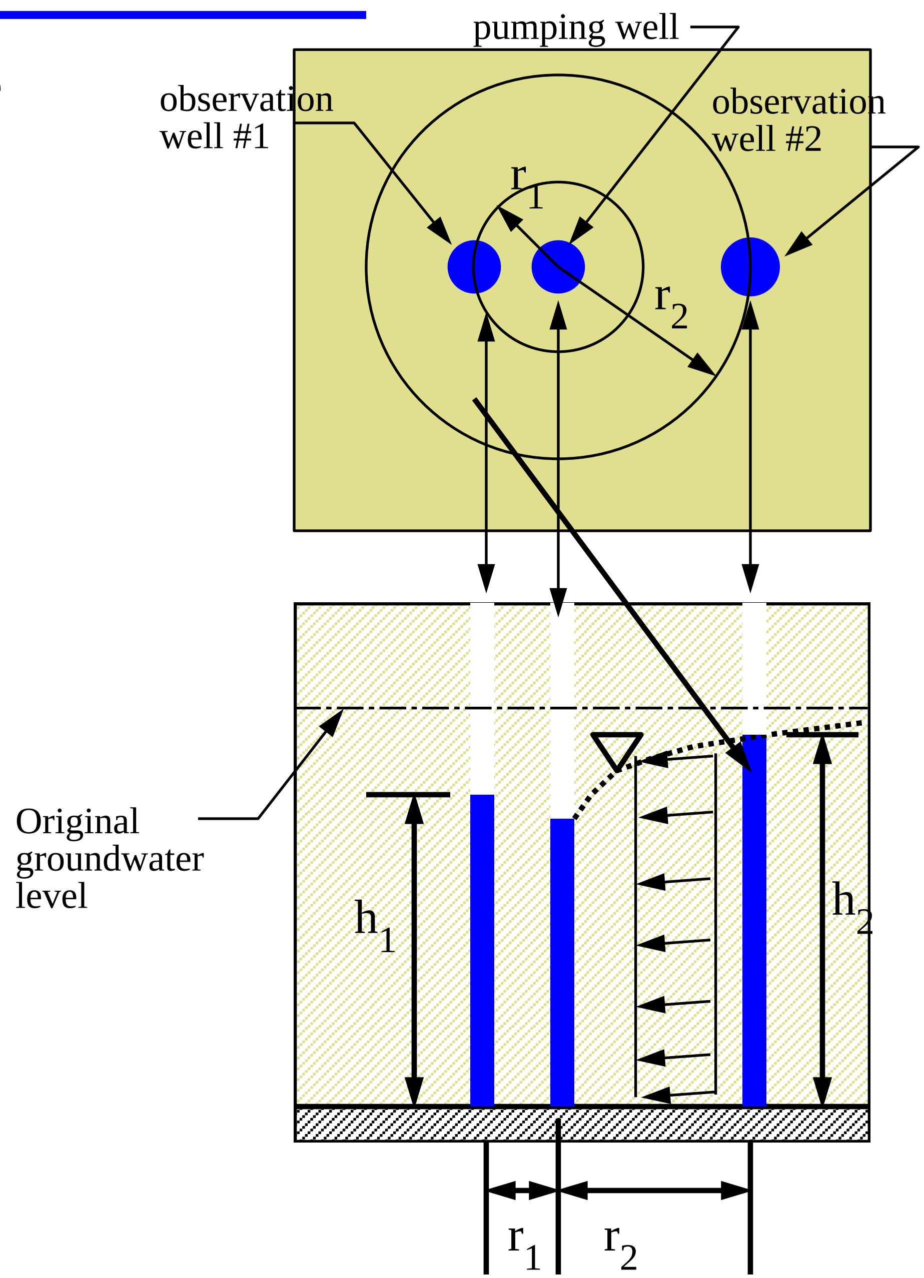
- |dh/dr| <<1 (i.e. the gradient is small)
- flow is therefore approximately horizontal
- homogeneous soil
- cylindrical symmetry about axis of pumping well

$q = 2\pi krh * dh/dr$ − First order ODE describing the radial flow rate into the pumping well.

Solution: $(\pi k/q)\,[h_2^2 - h_1^2] = \ln(r_2/r_1)$

Since q, r_1, r_2, h_1, h_2 can all be measured, we can solve for the soil's permeability k as:

$$k = (q/\pi)\,\ln(r_2/r_1)\,[h_2^2 - h_1^2]^{-1}$$



B. Hydraulic Conductivity Values for Soils:

- A *joint* property of both the soil and the fluid;
- Hydraulic conductivity k has units of (L/T) such as *m/s, ft/min, cm/day*, *etc.*

- Tabulated Hydraulic Conductivities (pore fluid is water)

SOIL TYPE	k(mm/sec)	Relative Permeability
coarse gravel, jointed rock	$> 10^{0}$	high
sand, fine sand	$10^{0} - 10^{-2}$	medium
silty sand, dirty sand	$10^{-2} - 10^{-4}$	low
silt, fine sandstone	$10^{-4} - 10^{-6}$	very low
clay, mudstone w/o joints	$< 10^{-6}$	impermeable

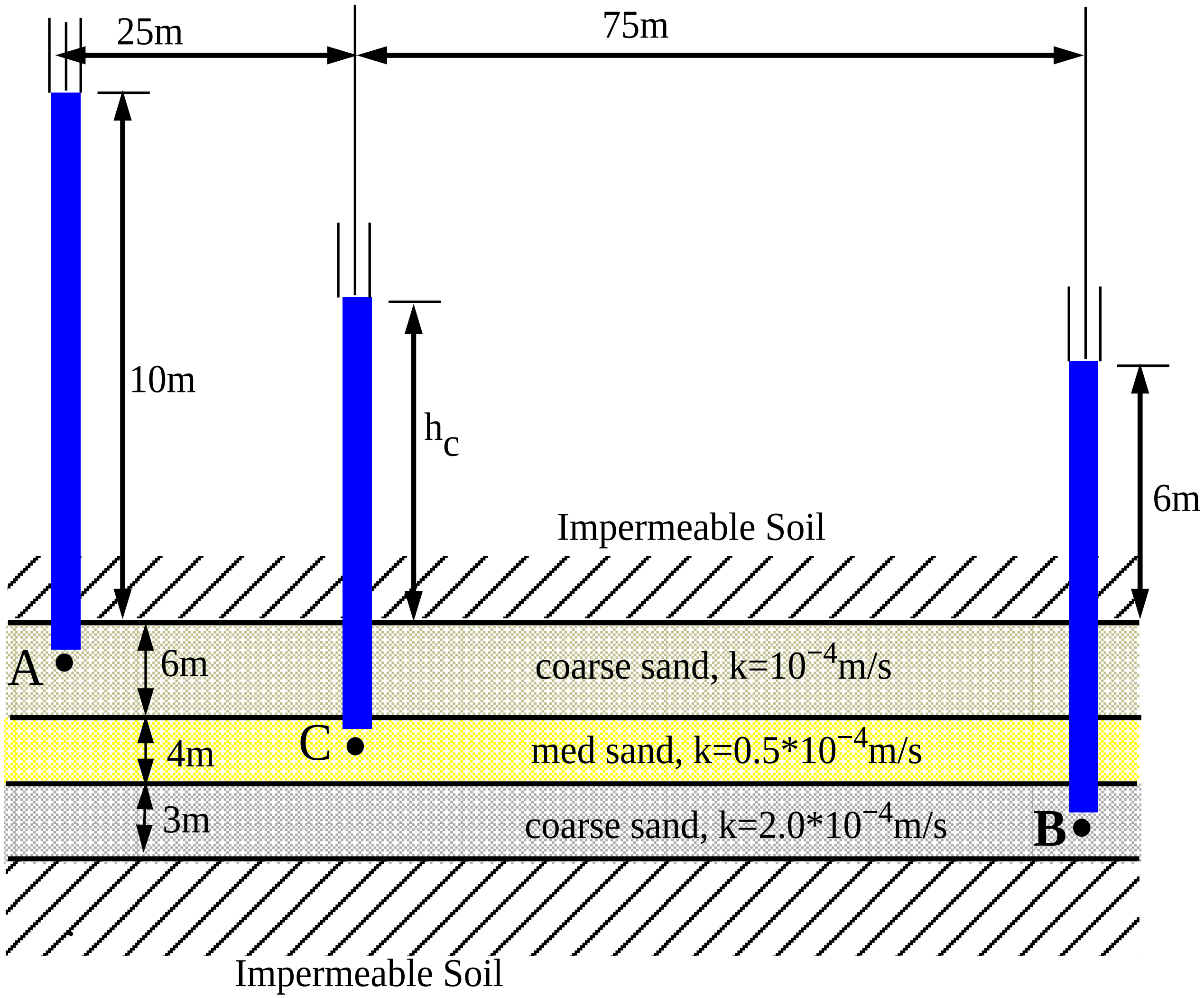
- Observation: As the grain size of the soil decreases, the conductivity decreases significantly. This is due to the higher SSA of fine−grained soils.

- Relationship between conductivity and fluid properties:

$k = K * [\gamma_f / \eta_f]$, where: K is the soil's absolute permeability (L^2);
γ_f is the unit weight of the fluid (FL^{-3}),
η_f is the viscosity of the fluid (FTL^{-2})

C. Effective Conductivities for Flows in Stratified Soils

1. Case #1: Steady Flow *Parallel* to Soil Layers
   - Assume that we wanted to compute the rate of horizontal fluid flow in this soil deposit. How would we do it?



- Since the soil layers are bounded above and below by impermeable layers, the flow can only be parallel to the soil layers.

- Since the head falls as we proceed from left to right, the water will also flow in this direction.

Given:
- The thickness of each layer (H_j)
- The conductivity of each layer (k_j)
- The hydraulic gradient in the soil deposit:
  - ♠ parallel to the layers, $i = \Delta h/L$
  - ♠ orthogonal to the layers, $i = 0$ (since there is no flow in that direction)

Solution:

Total flow = sum of flows in the layers :

$$q = \sum_{j=1}^{n} q_j = \sum_{j=1}^{n} v_j H_j = \sum_{j=1}^{n} k_j i_j H_j = i \sum_{j=1}^{n} k_j H_j$$

If we write an expression for the total flow as:

$q = k_{equiv} i\,H = i \sum_{j=1}^{n} k_j H_j$, then it is clear that

$k_{equiv} = (1/H) \sum_{j=1}^{n} k_j H_j$: **The effective conductivity of the layered soil**

Example Problem: Compute the flow rate in the three−layered medium on page 4.

Solution:

for the problem posed:

$i = 4m/100m = 0.04;$

$$k_{equiv} = (1/H)\sum_{j=1}^{n} k_j H_j$$

$$= (1/13m)[6m*10^{-4}m/s + 4m*0.5*10^{-4}m/s + 3m*2.0*10^{-4}m/s]$$

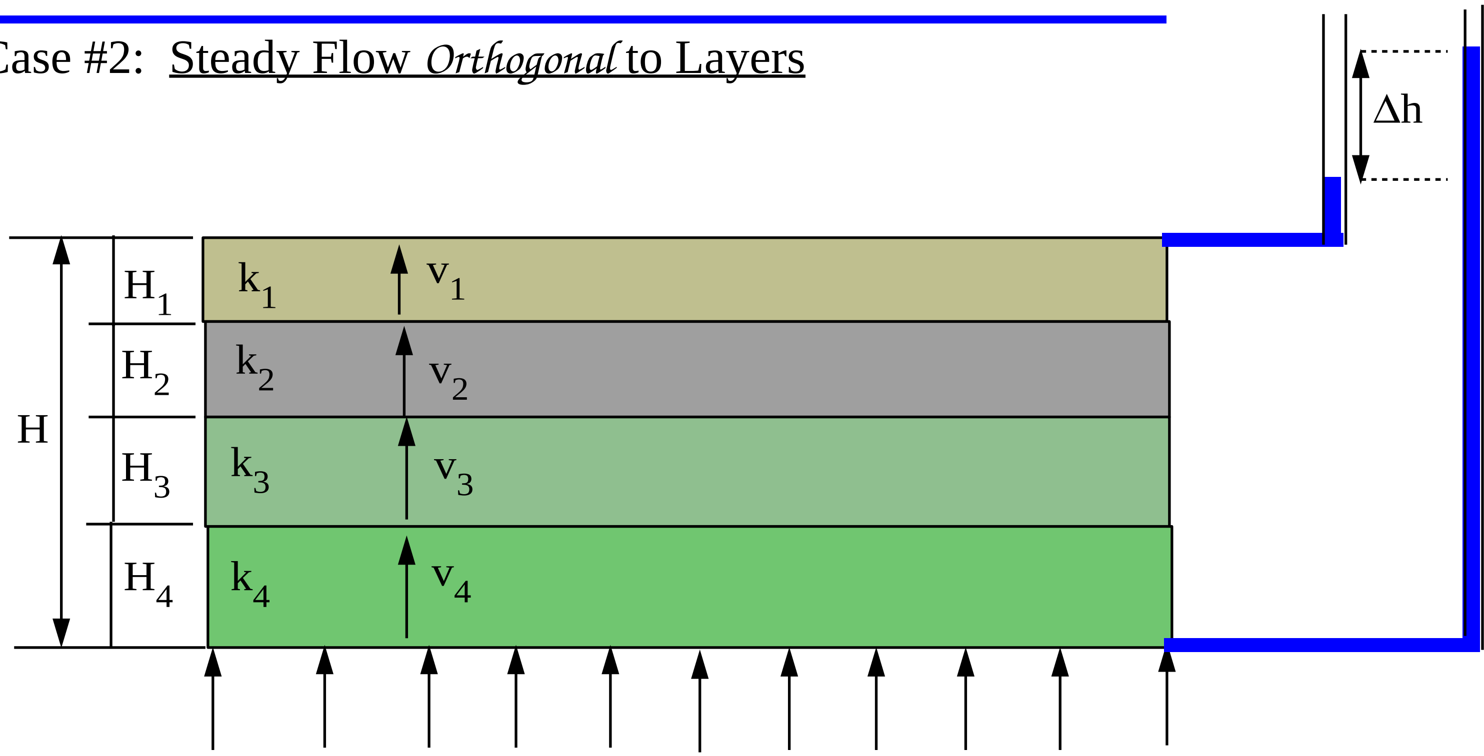
$$= 1.077*10^{-4}m/s$$

$$q = k_{equiv}\ i\ H$$

$$= (1.077*10^{-4}m/s)(0.04)(13m)$$

$$= 5.60*10^{-5}m^2/s$$

2. Case #2: Steady Flow *Orthogonal* to Layers



Observations:

- The total head loss across all of the layers is known to be Δh;
- The average hydraulic gradient $i = \Delta h/H$
- The thickness of each layer is H_j
- The conductivity of each layer is k_j

Also:

- From continuity considerations the discharge velocity in each layer must be the same. That is, $v_1 = v_2 = v_3 = v_4 = v$
- Darcy's Law holds in each layer:

  $v_j = k_j(\Delta h_j / H_j)$ where Δh_j is the head loss across the j^{th} layer.

Across the stratified soil deposit, we can write a form of Darcy's Law using the equivalent or effective conductivity of the layered soils:

$$v = k_{equiv} i = k_{equiv}(\Delta h/H)$$

where: k_{equiv} is the effective conductivity of the soil deposit;
Δh is the total head loss across the deposit;
H is the total thickness of the deposit.

Our objective is to find an expression for the equivalent or effective permeability.

Note that the total head loss across the deposit Δh is the sum of the head losses across all of the individual layers.

That is, $\Delta h = \sum_{j=1}^{n} \Delta h_j = \sum_{j=1}^{n} v_j H_j / k_j = v \sum_{j=1}^{n} (H_j/k_j)$

Now, recalling that $v = k_{equiv}(\Delta h/H) = k_{equiv}(v/H) \sum_{j=1}^{n} (H_j/k_j)$

Equating the first and last terms in the preceding expression and simplifying

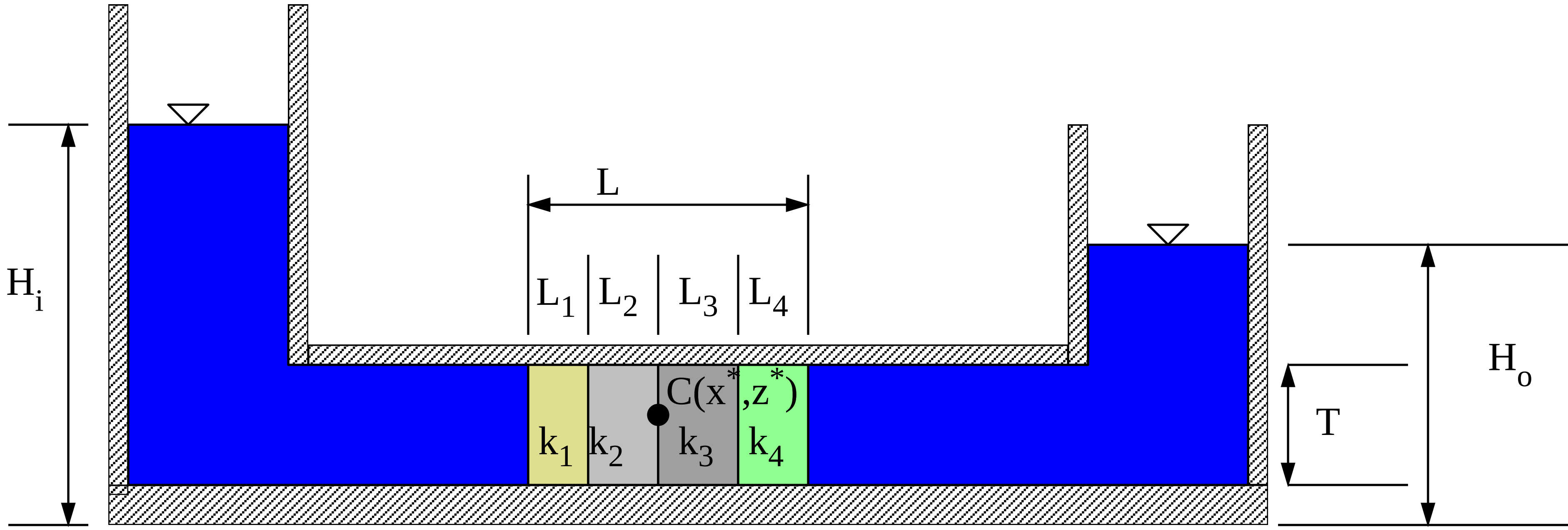
it thus follows that $k_{equiv} = H\left[\sum_{j=1}^{n} (H_j/k_j)\right]^{-1}$

Numerical Example:

Consider the stratified soil placed in the U−tube shown below.

Given: All soil properties and dimensions of the individual layers

Find: q, the flow rate through the soil (per unit width), and p_w at point C.



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3. Conclusions about stratified soils

- In general, for stratified soils, the $(k_{equiv})_{parallel}$ is not equal to $(k_{equiv})_{orthogonal}$.
- In cases where a soil deposit's permeabilities or conductivities are not the same in all directions, we saythat the properties are *anisotropic*.
- If the properties are the same in all directions, then it is said to be *isotropic*.
- Layered soil deposits typically have *anisotropic* effective or equivalent conductivities.

Very Important Points to Remember:

In fluid is not flowing in soil, the pore pressure can be computed directly using hydrostatics.

But when fluid is flowing in soils, we must first compute the value of head h and then compute pressure using the definition of head.