Period #24: Shear Strength of Soils (IV)

A. Review and Motivation

- The Mohr−Coulomb shear strength model for soils is of the form:

$|\tau_f| = c + \sigma'_n \tan(\phi_D)$ where:

$|\tau_f|$ is the absolute value of shear stress that causes failure on a given plane;
$c \geq 0$ is the cohesion of the soil;
σ'_n is the normal **effective** stress on a plane; and
$\phi_D \geq 0$ is the so−called **drained** angle of friction for a soil.

Typically, one does not expect to see a value of ϕ_D greater than about 45^o.

- Note that in the above shear strength criterion, the normal stresses are always effective stresses.
- For saturated, fine−grained soils subjected to loads, the effective stresses change over long periods of time as excess pressures dissipate.
- Therefore, the shear strength of a saturated, fine−grained soil will generally change (increase) with time as excess pore pressures dissipate.
- This period, we discuss different strength models for fine−grained soils that are based on combinations of total stresses and effective stress.
  - Typically, models based on effective stresses, apply to the long term drained behavior of fine−grained soils.
  - Models based on total stresses typically apply to short−term behaviors such as in the first few weeks/months after a structure is built upon a fine−grained soil deposit.

B. Drained Shear Strength Behavior of Clays

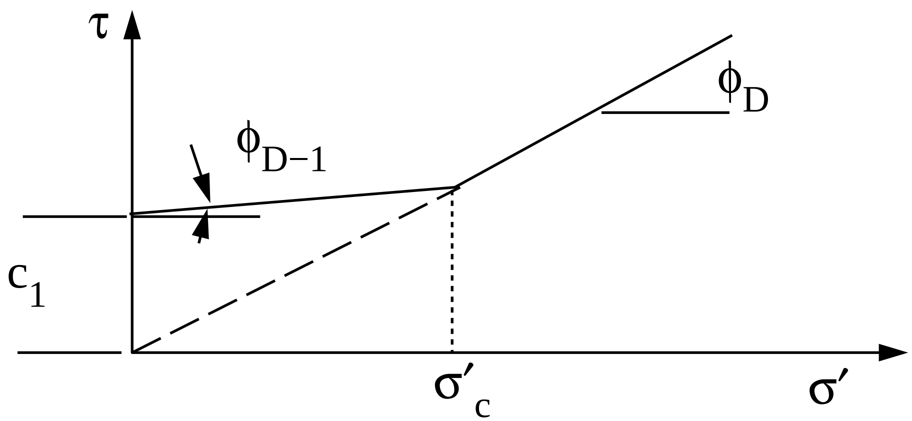
- For normally consolidated clays, $c \approx 0$, and the friction angle is denoted ϕ_D.
- For over−consolidated clays, $c > 0$, and the friction angle is denoted by ϕ_{D-1}.
- When the soil is overconsolidated: (or when $\sigma' < \sigma'_c$)

$$|\tau_f| = c_1 + \sigma'_n \tan(\phi_{D-1})$$

- When the soil is normally consolidated: (or when $\sigma' = \sigma'_c$)

$$|\tau_f| = \sigma'_n \tan(\phi_D)$$

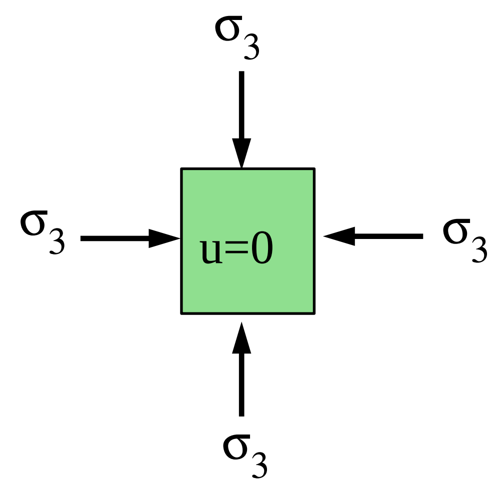
- Typical values of ϕ_D for silty/clayey soils: $12^o \le \phi_D \le 30^o$,
    - $\phi_D \approx 30^o$ for low plasticity soils (PI = 5−10);
    - $\phi_D \approx 12^o$ for high plasticity soils (PI = 50−100);
    - $12^o \le \phi_D \le 30^o$ for moderate plasticity soils ($10 \le PI \le 50$)



Example 24.1:

C. Drained and Undrained Strength Behaviors of Fine−Grained Soils

For simplicity, we will first discuss this in the context of triaxial compression tests, and then discuss the drained/undrained strength behaviors in terms of geotechnical applications.



1. Consolidated−Drained Triaxial Test (CD)

This test consists of two parts:

a) Hydrostatic consolidation:

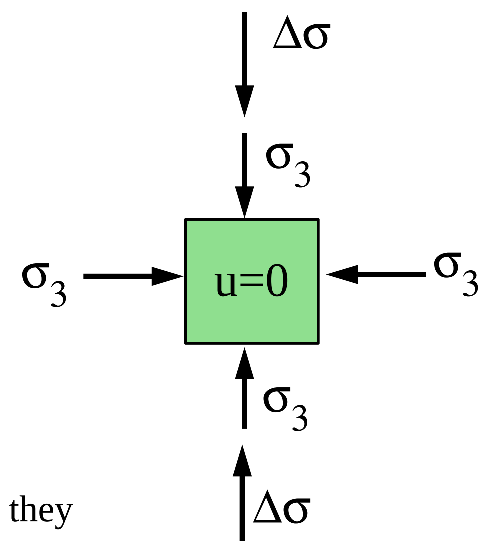
- The chamber pressure σ_3 is applied.
- Initially, this causes a buildup of pore pressure in the soil

$$u_c = B\sigma_3, \text{ where } B\sim1 \text{ for saturated soils.}$$

- The soil is allowed to drain/consolidate, following which $u_c = 0$.

b) Drained shear test:

- The axial load $\Delta\sigma$ is then applied so slowly, that any generated pore water pressures in the sample are permitted to dissipate.

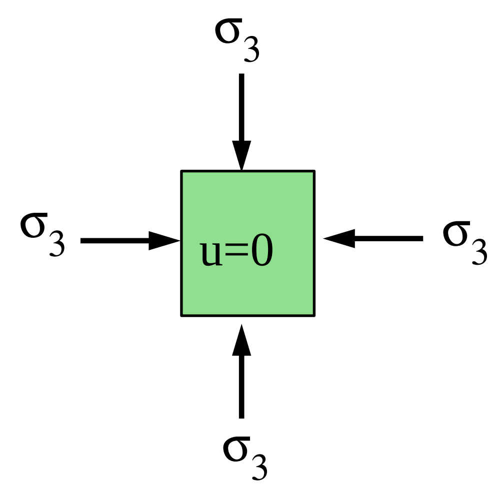


- In the consolidated drained (CD) triax test:

$$\sigma'_3 = \sigma_3 \quad \text{and} \quad \sigma'_1 = \sigma_3 + \Delta\sigma$$

- Because of the length of time required to perform CD triax tests, they are rarely performed for fine−grained soils such as silts and clays.

2. Consolidated−Undrained Triaxial Test (CU Triaxial Test)



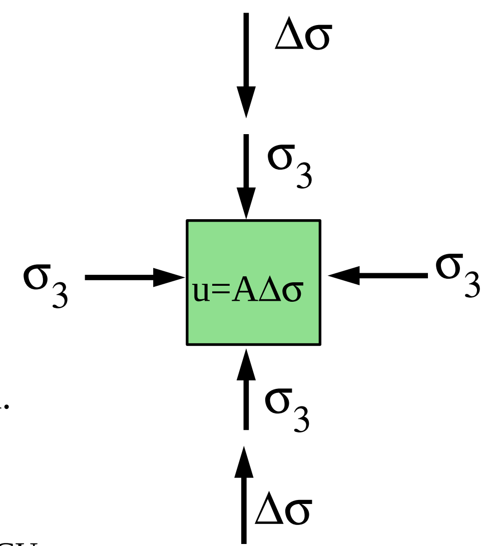
a) In this type of test, the soil is permitted to consolidate under the chamber confining pressure, just as in the CD Triax Test.

b) The second phase of this test involves **undrained shear.**

That is, pore pressures built up in the soil during application of $\Delta\sigma$ are not given time to dissipate. During this phase

$u = A\Delta\sigma$, where A is Skempton's pore pressure coeff.

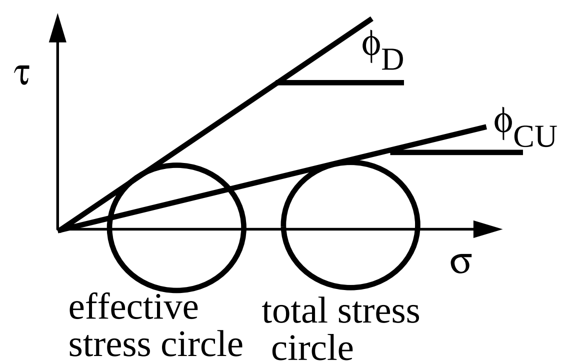
For NC soils, $0.5 < A < 1$
For OC soils, $0 < A < 0.5$



<u>Stresses during a CU Triax Test</u>

Total:	$\sigma_1 = \sigma_3 + \Delta\sigma$	$\sigma_3 = \sigma_3$	$\sigma_1 - \sigma_3 = \Delta\sigma$
Effective:	$\sigma'_1 = \sigma_3 + \Delta\sigma - u$	$\sigma'_3 = \sigma_3 - u$	$\sigma'_1 - \sigma'_3 = \Delta\sigma$

At shear failure of the soil:
- The effective stresses give ϕ_D, the drained angle of friction.
- The total stresses give ϕ_{CU}, the consolidate−undrained angle of friction.
- Typically ϕ_{CU} is about half of ϕ_D



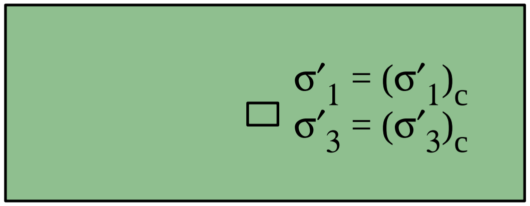
Example Problem 24.2:

3. Unconsolidated−Undrained (UU)Triax Test

- This test is also known as the quick (Q) shear test.
- It is typically performed on pre−consolidated silt/clay soil samples.

- A soil sample in the field is experiencing the following state of stresses:

  $\sigma'_1 = (\sigma'_1)_c$ and $\sigma'_3 = (\sigma'_3)_c$



- The procedure for performing this test is as follows:

  a) remove the soil sample from the field without disturbing it;

  b) place the soil in a triaxial testing chamber and increase the chamber pressure to a value $\sigma_3 = (\sigma_3')_c$. (Since the soil has already consolidated at this stress level, it does not need time to re−consolidate.)

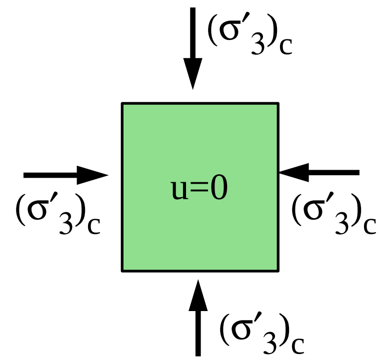
  c) increase the chamber pressure by an amount $\Delta\sigma_3$ without permitting consolidation under this added pressure;

  d) an increased axial stress $\Delta\sigma_d$ is then applied to the soil until failure occurs, with no drainage/consolidation permitted.

- We will now take a detailed look at what happens in the soil at each stage of this test.

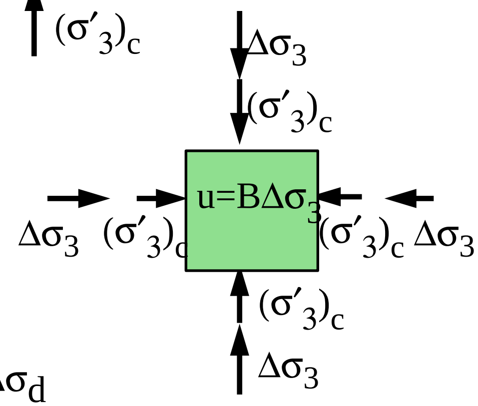
1) Re−application of the pre−consolidation stress:

$$\sigma_1 = \sigma'_1 = (\sigma'_3)_c$$
$$\sigma_3 = \sigma'_3 = (\sigma'_3)_c$$
$$u = 0$$



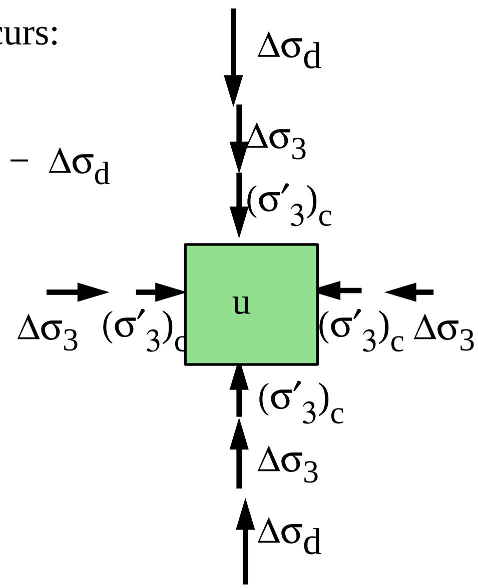
2) Increase of the chamber confining pressure by $\Delta\sigma_3$
[no drainage permitted]

$$\sigma_1 = (\sigma_3')_c + \Delta\sigma_3 \qquad \sigma'_1 = (\sigma'_3)_c$$
$$\sigma_3 = (\sigma'_3)_c + \Delta\sigma_3 \qquad \sigma'_3 = (\sigma'_3)_c$$
$$u = B\,\Delta\sigma_3 \approx \Delta\sigma_3$$



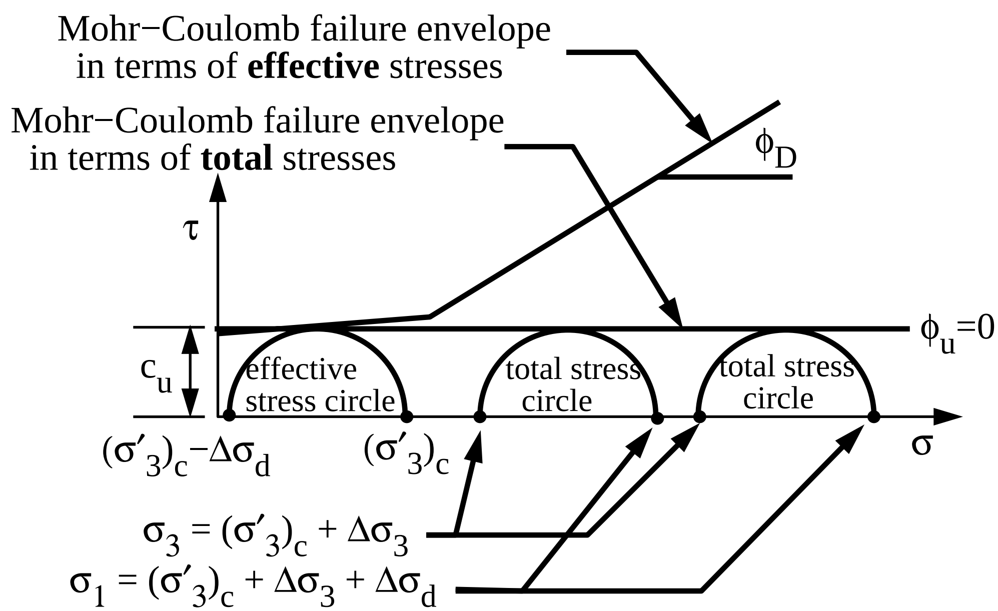
3) Increase the axial load by $\Delta\sigma_d$ until failure occurs:

$$\sigma_1 = (\sigma'_3)_c + \Delta\sigma_3 + \Delta\sigma_d \qquad \sigma'_1 = (\sigma'_3)_c$$
$$\sigma_3 = (\sigma'_3)_c + \Delta\sigma_3 \qquad \sigma_3' = (\sigma'_3)_c - \Delta\sigma_d$$
$$u = B\,\Delta\sigma_3 + A\,\Delta\sigma_d$$
$$\approx \Delta\sigma_3 + \Delta\sigma_d$$



Test Summary:

Loading	Total stresses	Neutral stresses	Effective stresses
Overburden	$\sigma_1 = (\sigma'_3)_c$ $\sigma_3 = (\sigma'_3)_c$	$u = 0$	$\sigma'_1 = (\sigma'_3)_c$ $\sigma'_3 = (\sigma'_3)_c$
Added continement	$\sigma_1 = (\sigma'_3)_c + \Delta\sigma_3$ $\sigma_3 = (\sigma'_3)_c + \Delta\sigma_3$	$u = \Delta\sigma_3$	$\sigma'_1 = (\sigma'_3)_c$ $\sigma'_3 = (\sigma'_3)_c$
Axial loading	$\sigma_1 = (\sigma'_3)_c + \Delta\sigma_3 + \Delta\sigma_d$ $\sigma_3 = (\sigma'_3)_c + \Delta\sigma_3$	$u = \Delta\sigma_3 + \Delta\sigma_d$	$\sigma'_1 = (\sigma'_3)_c$ $\sigma'_3 = (\sigma'_3)_c - \Delta\sigma_d$



- For the unconsolidated, un−drained test:
  - c_u is the undrained cohesion of the soil (= $\Delta\sigma_d/2$).
  - ϕ_u is the undrained angle of friction (~ 0).

- **Observe that no matter how much the added confining pressure $\Delta\sigma_3$ is increased in this test, the measured shear strength of the soil will be about the same.**

D. Summary and Observations:

- Physically, the shear failure behavior in soils is governed by the relations between **normal effective stresses** and shear stresses.

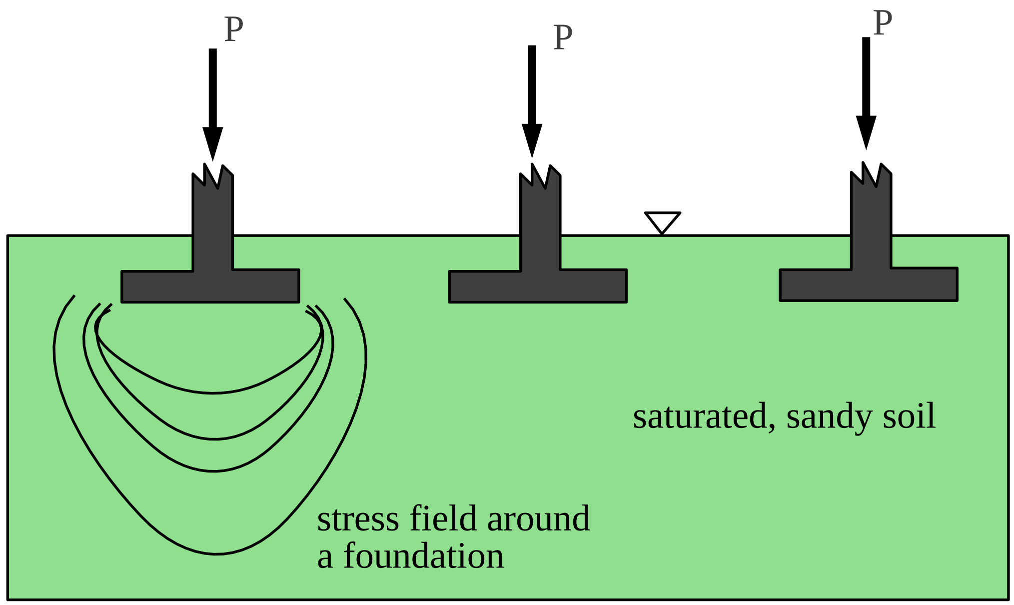
- In principle, we should only need to use the Mohr−Coulomb envelope defined with respect to effective stresses.

- In geotechnical engineering practice, however, there are many cases where we do not know the neutral stresses in the soil. Thus we only know the total stresses and not the effective stresses.

- For these special cases (which typically involve undrained behaviors of fine−grained soils) we use a Mohr−Coulomb failure envelope in terms of total stresses.

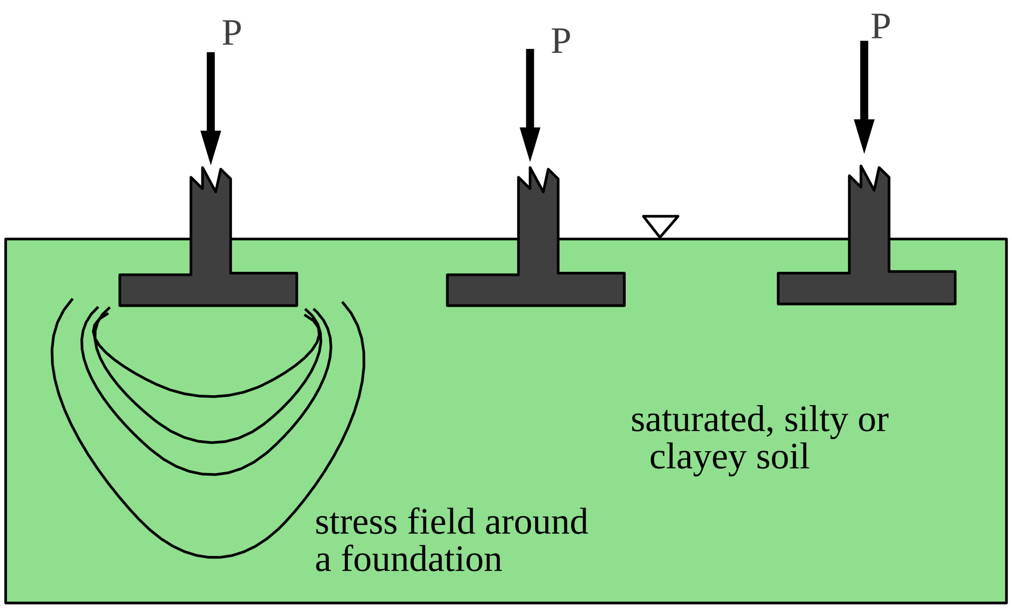
Example:

- Assume that we are applying foundation loads to a saturated sandy medium.
  - Since the stresses from the foundation loads are quickly transferred to the soil skeleton, the foundation loads are carried by effective stresses.
  - To determine whether or not shear failure would occur in the soil from the foundation loads, we would use a **drained** shear strength criterion with respect to the **effective stresses** in the soil mass.



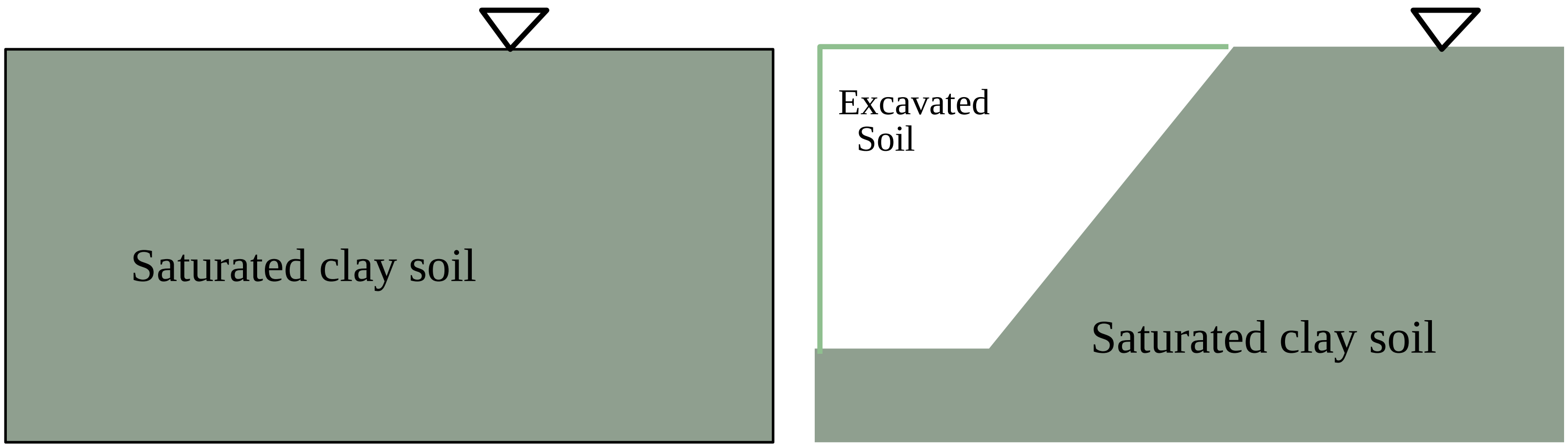
Example:

- Assume that we are applying the same foundation loads to a saturated clay or silt soil.
    - In the short term the increased stresses from the foundation loads are quickly transferred to the soil skeleton **and** the pore fluid. In the **short term**, we would use an **undrained shear strength failure criterion ($c=c_u$, $\phi=0$)** to assess possible shear failure.
    - In the long term, the increased stresses from the foundation loads are carried by the soil skeleton via effective stresses. In the **long term**, we would use a **drained shear strength** failure criterion to assess possible shear failure.



Example:

- Assume that we are quickly *cutting* a slope in a saturated clay soil deposit.
  - In the short term, we would use an **undrained shear strength** model to determine whether or not shear failure (or a slope failure) would occur.
  - In the long term, we would use a **drained shear strength** model to determine whether or not shear failure (or a slope failure) would occur.



•Final Thoughts on Shear Strengths:

The major differences in shear strength behaviors of clays/silts and sands/gravels can be traced back to grain size effects:

Fine−grained soils

high SSA's

low permeabilities (drain slowly)

relevant strength is typically **undrained**

Coarse−grained soils

low SSA's

high permeabilities (drain quickly)

relevant shear strength is is typically **drained**