Period #18: Stresses in the Soil Mass (I)

A. Motivation:

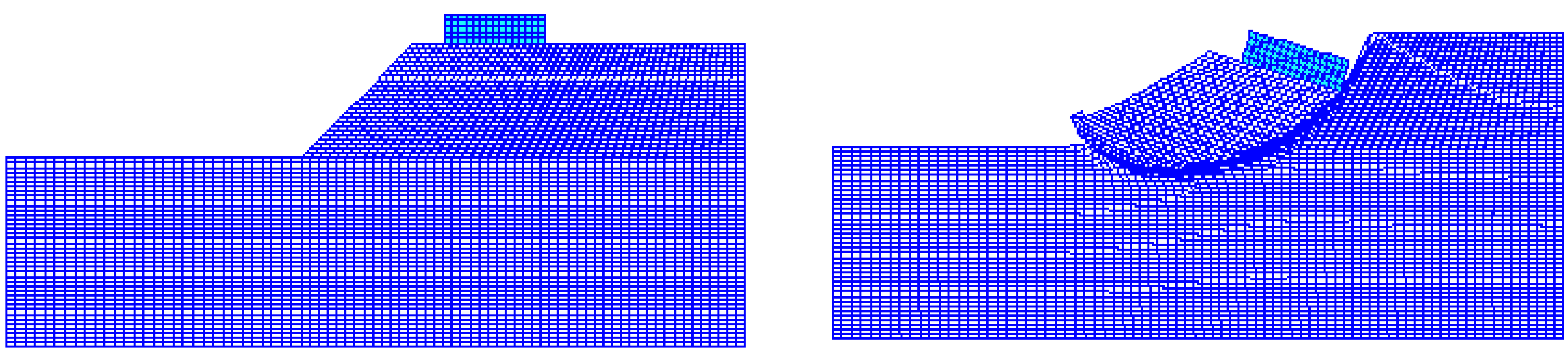
- For the next two class periods, we will discuss how to compute stresses in soils under a variety of loadings.

- This is a very practical issue, since:

  a) Consolidation settlements in soils are strongly related to increased stresses from applied loads.

  b) Shear failure in soils can occur due to applied loads, and as engineers, we need to be able to predict whether or not failure will occur.

- For example, consider the model figure shown below which simulates a building constructed atop a soil slope. The combined internal stresses in the soil can be sufficiently large to cause shear failure with potentially severe consequences.



- Discussion on stresses in soils will be divided into three parts:

  1) Usage of Mohr's Circle to compute stresses at a point;

  2) Usage of Elasticity to compute spatial distributions of stresses from applied loadings;

  3) Applications.

B. Mohr's Circle:

- At a given point A in the soil mass, we are given the state of stresses in two−dimensions with respect to a set of x,y coordinate axes.

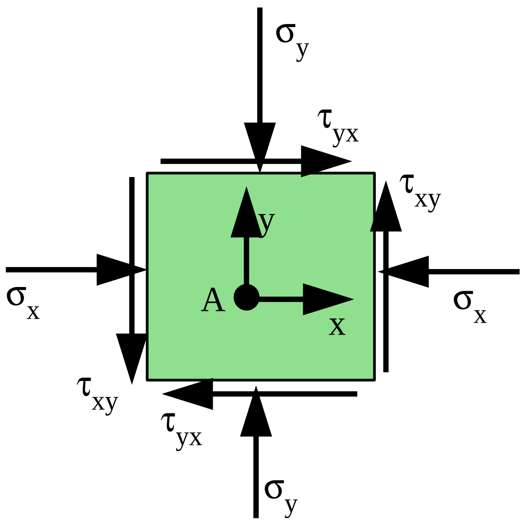
- Meaning of stress components:

σ_y is the normal stress on a plane having a unit normal in the y−direction;

σ_x is the normal stress on a plane having a unit normal in the x−direction;

τ_{xy} is the y−directed shear stress on a plane having a unit normal in the x−direction;

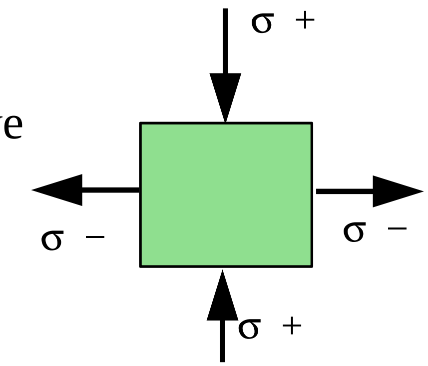
τ_{yx} is the x−directed shear stress on a plane having a unit normal in the y−direction;



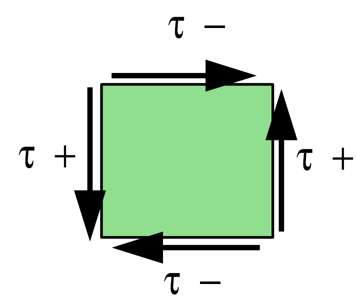
- While the values of τ_{xy} and τ_{yx} will be different, their magnitudes will always be the same.

Mohr's Circle Sign Conventions:

- Compressive normal stresses are positive

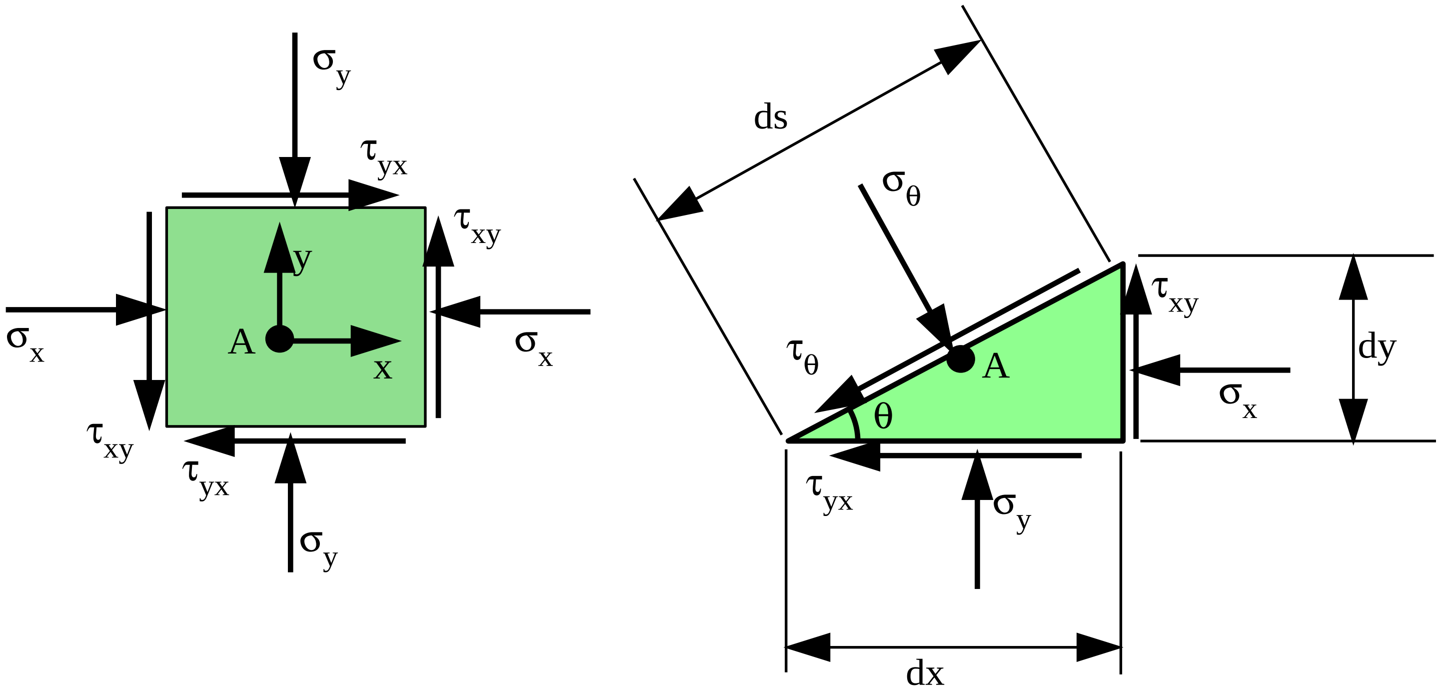


- Shear stresses are positive, if when they act on two opposing faces, they tend to produce a counterclockwise rotation.



- The question that Mohr's Circle helps to answer is:

Given the stresses on any two perpendicular planes passing through a point A, what are the stresses on all other planes passing through the same point?



- For example, assume that we want to compute the stresses on the plane passing through point A that makes a counterclockwise angle θ with the plane having a unit normal in the −y direction.

- Using simple statics for the wedge of soil shown, we can write the force equilibrium equations in the x− and y−directions as:

$$\Sigma F_x = 0 = -\sigma_x\, ds\, \sin(\theta) - \tau_{yx}\, ds\, \cos(\theta) - \tau_\theta\, ds\, \cos(\theta) + \sigma_\theta\, ds\, \sin(\theta)$$
$$\Sigma F_y = 0 = \sigma_y\, ds\, \cos(\theta) + \tau_{xy}\, ds\, \sin(\theta) - \tau_\theta\, ds\, \sin(\theta) - \sigma_\theta\, ds\, \cos(\theta)$$

- Solving for σ_θ and τ_θ we can find:

$$\sigma_\theta = \sigma_x \sin^2(\theta) + \sigma_y \cos^2(\theta) + 2\,\tau_{xy} \sin(\theta)\cos(\theta)$$
$$\tau_\theta = (\sigma_y - \sigma_x)\sin(\theta)\cos(\theta) + \tau_{xy}(\sin^2(\theta) - \cos^2(\theta))$$

- Using a few trigonometric identities and some algebra (see the text) it can be shown that the points σ_θ and τ_θ lie on a circle plotted on the (σ, τ) axes.

$$\sigma_\theta = c + a\cos(2\theta) + b\sin(2\theta)$$
$$\tau_\theta = a\sin(2\theta) - b\cos(2\theta)$$

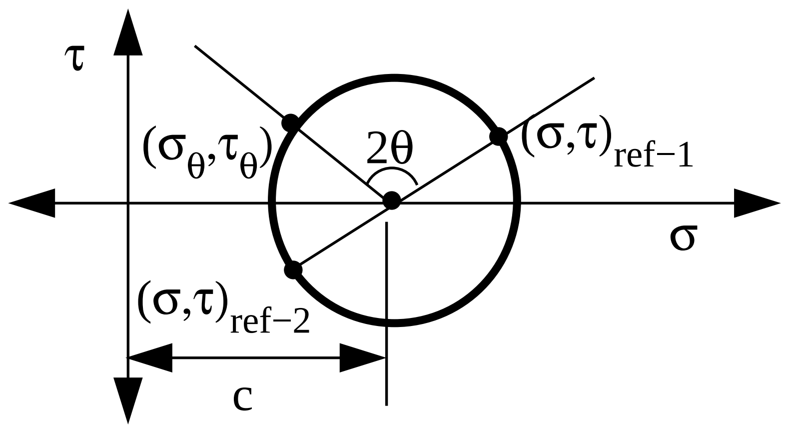
where: c = the center of the circle on the σ axis

$= \frac{1}{2}(\sigma_x + \sigma_y)$

$a = \frac{1}{2}(\sigma_x - \sigma_y)$

$b = \tau_{xy}$

$r = [a^2 + b^2]^{1/2}$



- Mohr's Circle Observations:

1) The largest possible normal stress on any plane is the major principal stress σ_1

$$\sigma_1 = c + r = \tfrac{1}{2}(\sigma_x + \sigma_y) + [\ \{\tfrac{1}{2}(\sigma_x - \sigma_y)\}^2 + \tau_{xy}^2\ ]^{1/2}$$

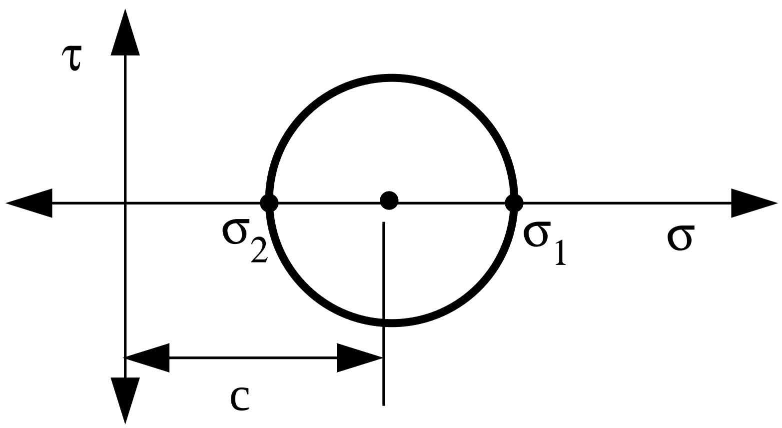
2) The smallest possible normal stress on any plane is the minor principal stress σ_2

$$\sigma_2 = c - r = \tfrac{1}{2}(\sigma_x + \sigma_y) - [\ \{\tfrac{1}{2}(\sigma_x - \sigma_y)\}^2 + \tau_{xy}^2\ ]^{1/2}$$

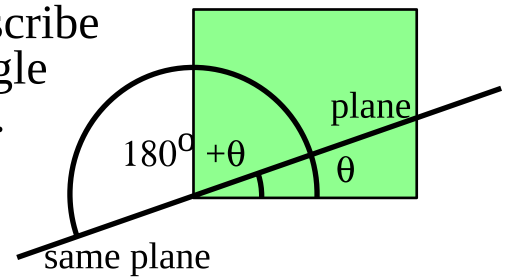
3) Shear stresses τ vanish on the principal planes.

4) The largest possible shear stress on any plane,

$$\tau_{max} = r = [\ \{\tfrac{1}{2}(\sigma_x - \sigma_y)\}^2 + \tau_{xy}^2\ ]^{1/2}$$

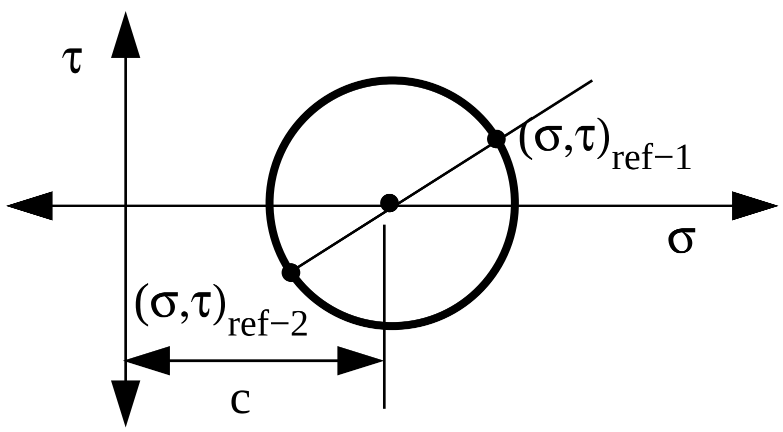


5) There are 360° in a circle, yet only 180° to describe the orientation of planes. Thus a physical angle change of θ is represented as 2θ on the circle.



6) The sum of normal stresses on any two perpendicular planes is always equal to 2c.

7) The stresses on the two perpendicular reference planes will always plot as two diametrically opposed points on a Mohr's Circle.



C. Brief Examples:

Two brief examples were presented here.

D. The Pole Method

Given the stresses on two perpendicular reference planes, there are *at least* three ways to find the stresses making an angle θ with one of the planes:

1) Use the circle formulas derived;

2) Use Mohr's Circle with an interior angle approach; or

3) Use Mohr's Circle with **the Pole Method**. <u>This is the preferred method which you will be expected to master in this course.</u>

Pole Method for finding stresses on a plane any orientation:

a) Find the stresses on two perpendicular reference planes. (For notational simplicity, it is assumed they are aligned with the x and y axes, but this is not required.

b) Find the center c, and the radius r of the circle.

c) Draw the circle.

d) To locate the pole P:

1) Through the point representing the stresses on the first reference plane (x−plane), draw the orientation of the first reference plane (x−plane is vertical).

2) The point where this line intersects the Mohr's Circle is the pole P.

e) **To find the stresses on a plane of any orientation:**

1) draw a line through the pole P parallel to the plane;
2) the point where this line intersects the Mohr's circle gives the stresses (σ,τ) on the plane of interest.

Example Problem #1 Using Pole Method:

Example Problem #2 Using Pole Method: