Spring Semester 2008
53:086 Civil Engineering Materials
Department of Civil & Environmental Engineering
The University of Iowa

Assignment #7 Due: Thursday 4/08/2008

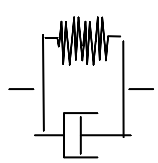
Textbook Problems: Chapter 9: 1,2,5,7,10,11 (50%)

Additional Problem on Viscoelasticity: (50%)
Consider an asphalt cement with a shear modulus G of 5 MPa, and a relaxation frequency f_r of 0.1Hz. The asphalt undergoes cyclic shear stress loading of 0.1 MPa in a dynamic shear rheometer. That is, $\tau = \tau_0 e^{i\omega t}$ where $i = \sqrt{-1}$; and $\tau_0 = 0.1\,MPa$.

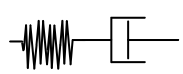
1. Assuming the asphalt's constitutive behavior is governed by the Voigt model presented above, plot the stress-strain (τ vs. γ) hysteresis curves for the asphalt for loading frequencies of:
   a. ω = 0.1 rad/sec;
   b. ω = 1.0 rad/sec;
   c. ω = 10 rad/sec;
   d. ω = 100 rad/sec;
2. Similarly, assuming the Maxwell model presented above governs the asphalt's constitutive behavior, plot stress-strain hysteresis curves for the same loading frequencies.
3. Discuss your results. How do the Voigt and Maxwell models differ? How do the cyclic stress-strain behaviors change with frequency?

Background Information

There are numerous ways to model viscoelastic materials like asphalt. Two of the simplest are the Voigt and Maxwell models of viscoelasticity. Both models feature an elastic spring of stiffness G which represents linear elastic behavior and a dashpot of viscosity η which represents viscous behavior. In the Voigt model, the spring and dashpot are arranged in parallel, whereas in the Maxwell model, they're arranged in series. Below, we develop constitutive models for both models.



a) Voigt model



b) Maxwell model

Voigt Model

For spring and dashpot members arranged in parallel, the assumption is that they both have the same strain, although the force in each will generally be different. Assume that the strain in the spring is γ_s, it follows that the stress in the spring is $\tau_s = G\gamma_s$. In the dashpot, the stress is given as follows: $\tau_d = \eta\dot{\gamma}_d$. Since the strain and strain rates in the spring are identical, and equivalent to those of the entire unit, and since the overall stress of the unit is the sum of those in the spring and dashpot, we get the following first order ODE that governs the Voigt model: $\tau = \tau_s + \tau_d = G\gamma + \eta\dot{\gamma}$

Maxwell Model

For spring and dashpot members arranged in series, the assumption is that they both have the same stress, although the strain in each will generally be different. Assuming again that the strain in the spring is γ_s, it follows that the stress in the spring is $\tau_s = G\gamma_s$. The incremental form of this same relation is $\dot{\tau}_s = G\dot{\gamma}_s$. In the dashpot, the stress is given as follows: $\tau_d = \eta\dot{\gamma}_d$. Since the stress rates in the spring and dashpot identical, and equivalent to those of the entire unit, and since the overall strain of the unit is the sum of those in the spring and dashpot, we get the following first order ODE that governs the Maxwell model: $\dot{\gamma} = \dot{\gamma}_s + \dot{\gamma}_d = \dot{\tau}/G + \tau/\eta$. Upon re-arranging first order ODE governing the Maxwell model, the following is obtained: $\dot{\tau} + f_r \cdot \tau = G\dot{\gamma}$. In the preceding equation, $f_r = G/\eta$ is the so-called relaxation frequency.

Hints:

The stress history in the material is prescribed $\tau = \tau_0 e^{i\omega t}$. The resulting strain history in the material will be $\gamma = \gamma_0 e^{i(\omega t + \delta)}$. For each material model and loading frequency, you'll need to solve for γ_0 the strain magnitude, and δ the phase angle between the stress and strain. Once you solve for these, obtain $\tau(t)$ and $\gamma(t)$ for $t \in [0, T]$. Then for a sequence of discrete time values, plot the current stress and strain on a plot. You'll definitely want to do this in Excel. Your results for the Voigt model should look similar to what is shown in the figure below.