Period #11 Notes: MECHANICS OF PORTLAND CEMENT CONCRETE

A. Brief Overview

Portland cement concrete is a composite material consisting of aggregate particles and the hydrated cement paste matrix that binds them together. The mass density, stiffness, and strength properties of portland cement concrete (pcc) are thus functionally dependent on the properties of both the matrix material (i.e. the hcp) and the reinforcement material (the aggregate). In this course, we study at least three composite material systems: (1) pcc; (2) asphalt cement concrete (acc); and (3) fiber-reinforced plastics (FRPs). Since pcc is the first composite material on this list, we'll have to begin by introducing some basic ideas that apply to all composite materials.

B. Mass Density and Volume Fractions

When heavy materials are used in composites, the mass density of the composite increases, and vice versa.

Consider the schematic of the composite shown in Fig. 11.1 that consists of two materials:

(1) a particulate reinforcing phase (r); and

(2) the continuous matrix phase (m).

The total volume of the composite sample shown can be decomposed into that of the particulate reinforcing phase, and that of the continuous matrix phase.

$$V = V_r + V_m \qquad (11.1)$$

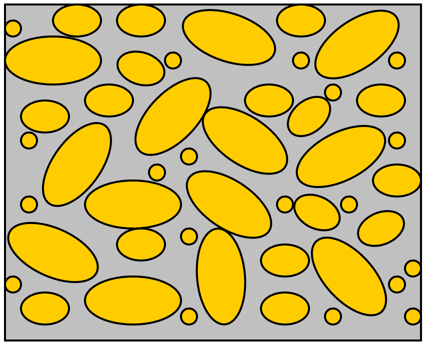

Fig. 11.1. Representative volume element of a two-phased composite material.

If one takes the volume equation (11.1) and divides through by the total volume V, the following equation results:

$$\frac{V}{V} = \frac{V_r}{V} + \frac{V_m}{V} \Rightarrow 1 = \phi_r + \phi_m \qquad (11.2a)$$

$$\text{where:} \quad \phi_r = \frac{V_r}{V} \quad \text{(the reinforcement volume fraction)} \qquad (11.2b)$$

$$\phi_m = \frac{V_m}{V} \quad \text{(the matrix volume fraction)} \qquad (11.2c)$$

It is common to discuss the composition of composite materials in terms of the <u>volume fractions</u> of the reinforcing phase and the matrix phase.

Generally in pcc, the aggregate volume fraction is between 60% and 80%, and the matrix volume fraction between 20% and 40%.

The total mass of a composite material is the sum of the masses of the reinforcement and matrix phases.

$$M = M_r + M_m \qquad (11.3a)$$

$$= \rho_{\mathrm{r}} V_r + \rho_{\mathrm{m}} V_m \qquad (11.3b)$$

The mass density of the composite is the total mass per total unit volume:

$$\rho = \frac{M}{V} = \rho_{\mathrm{r}} \phi_r + \rho_{\mathrm{m}} \phi_m \qquad (11.4)$$

C. Effective Stiffness of Composites

While the effective mass density of a composite is directly expressed by (11.4), it is much less straightforward to express the effective stiffness of a composite in terms of the stiffnesses and volume fractions of the constituent phases. For this reason, there are a variety of different models and assumptions that can be used to **estimate** the effective stiffness (or elastic moduli) of composites. Three models that will be presented here are:

1. The Voigt isostrain rule of mixtures;
2. The Reuss isostress rule of mixtures; and
3. The hybrid rule of mixtures.

1. The Voigt rule of mixtures

To understand the isostrain rule of mixtures, consider a representative volume element of the composite where the matrix and reinforcement phases have been separated into distinct regions as shown in Fig. 11.2.

Now assume that the composite is subjected to a one-dimensional stress loading of magnitude σ as shown.

Arranged and loaded as shown, the matrix and reinforcement phases are being loaded in parallel.

Loaded in parallel like this, the strain in both phases should be the same.

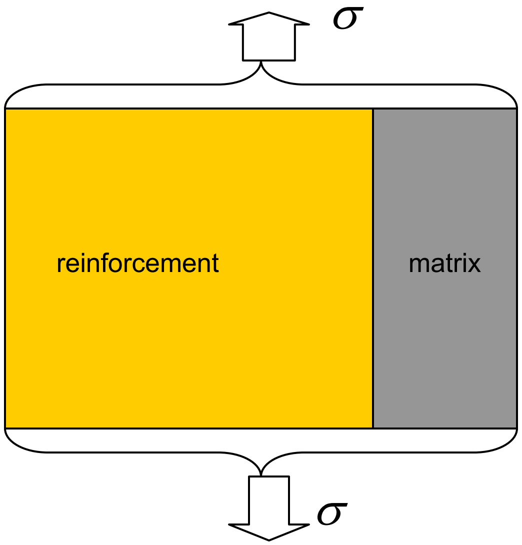

Fig. 11.2. Composite grouped into separate materials and loaded parallel to material allignment.

If the strain in both the matrix ε_m and reinforcement ε_r phases is the same, then the overall strain of the composite is also the same.

$$\varepsilon = \varepsilon_r = \varepsilon_m \tag{11.5}$$

Assuming linear elastic behavior, the stress in each phase would be consistent with its Young's modulus:

$$\sigma_r = E_r * \varepsilon_r = E_r * \varepsilon \qquad \text{In the reinforcing phase} \tag{11.6a}$$

$$\sigma_m = E_m * \varepsilon_m = E_m * \varepsilon \qquad \text{In the matrix phase} \tag{11.6b}$$

The forces carried in the reinforcing and matrix phases, respectively, are:

$$f_r = E_r * \varepsilon * A_r = E_r * \varepsilon * \phi_r * A \qquad \text{reinforcing phase} \tag{11.7a}$$

$$f_m = E_m * \varepsilon * A_m = E_m * \varepsilon * \phi_m * A \qquad \text{matrix phase} \tag{11.7b}$$

The average stress in the composite is the total force per gross cross-sectional area:

$$f = f_m + f_r \tag{11.8}$$

$$\sigma = \frac{f}{A} = \phi_m * E_m * \varepsilon + \phi_r * E_r * \varepsilon \tag{11.9a}$$

$$= \left(\phi_m * E_m + \phi_r * E_r\right) * \varepsilon \tag{11.9b}$$

$$= \mathrm{E}^*_{\mathrm{voigt}} * \varepsilon \tag{11.9c}$$

So, when the material phases of a composite are oriented in parallel with a one-dimensional loading, the strain in both phases is the same, and the effective stiffness of the composite is the weighted average of the two individual phase moduli.

Restated:

$$E^*_{voigt} = \phi_m E_m + \phi_r E_r \qquad (11.10)$$

The Voigt-isostrain rule of mixtures provides an upper-bound on the Young's modulus of a composite with phases *r* and *m*.

2. *The Reuss Isostress Rule of Mixtures*

To understand the isostress rule of mixtures, consider a representative volume element of the composite where the matrix and reinforcement phases have been separated into distinct regions as shown in Fig. 11.3.

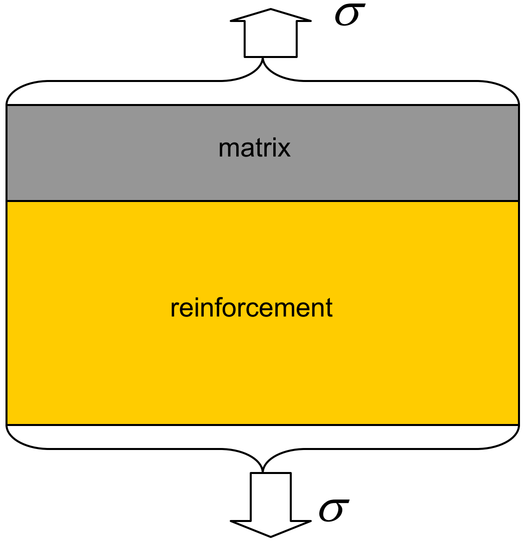

Fig. 11.3. Composite grouped into separate materials and loaded orthogonal to material allignment.

Now assume that the composite is again Subjected to a one-dimensional stress loading of magnitude σ as shown.

Loaded in series like this, the stress in both phases should be the same and equal to the overall applied stress σ.

$$\sigma = \sigma_r = \sigma_m \qquad (11.11)$$

The strain in each phase is as follows:

$$\varepsilon_r = \frac{\sigma}{E_r}; \quad \text{and} \quad \varepsilon_m = \frac{\sigma}{E_m} \qquad (11.12)$$

The total change in length of the specimen is as follows:

$$\Delta L_r = \varepsilon_r * L_r = \frac{\sigma}{E_r} * \phi_r * L \qquad \text{reinforcing phase} \qquad (11.13a)$$

$$\Delta L_m = \varepsilon_m * L_m = \frac{\sigma}{E_m} * \phi_m * L \qquad \text{matrix phase} \qquad (11.13b)$$

The overall strain of the specimen is the total change in length divided by the original length:

$$\varepsilon = \frac{\Delta L}{L} = \frac{\Delta L_r + \Delta L_m}{L} = \frac{\frac{\sigma}{\mathrm{E_r}}\phi_r L + \frac{\sigma}{\mathrm{E_m}}\phi_m L}{L} \qquad (11.14a)$$

$$= \left(\frac{\phi_r}{\mathrm{E_r}} + \frac{\phi_m}{\mathrm{E_m}}\right)\sigma \qquad (11.14b)$$

$$\sigma = \left(\frac{\phi_r}{\mathrm{E_r}} + \frac{\phi_m}{\mathrm{E_m}}\right)^{-1}\varepsilon \qquad (11.15)$$

Thus, the effective Young's modulus of a composite in accordance with the Reuss isostress assumption is:

$$E^*_{reuss} = \left(\frac{\phi_r}{\mathrm{E_r}} + \frac{\phi_m}{\mathrm{E_m}}\right)^{-1} \qquad (11.16)$$

Just as the Voigt isostrain rule of mixtures gives an upper bound on composite stiffness, the Reuss isostress rule of mixtures gives a lower bound on composite stiffness.

3. **The Hybrid Rule of Mixtures**

The hybrid rule of mixtures is based on the assumption that the reinforcement is embedded within the matrix as shown in Fig. 11.4.

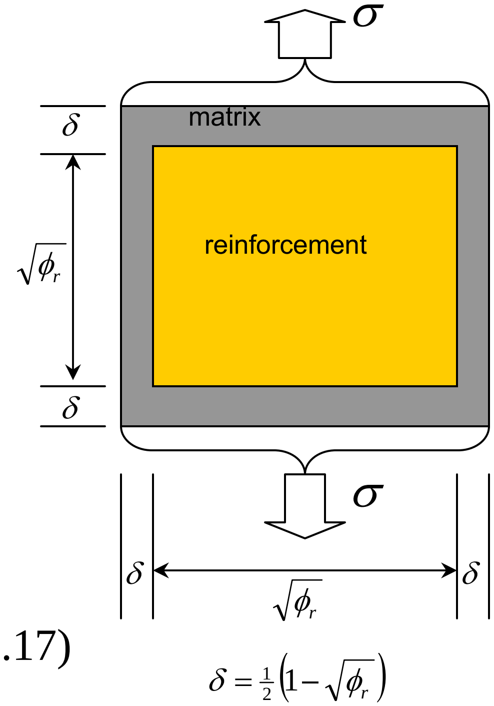

Fig. 11.4. Composite grouped into separate materials for the hybrid rule of mixtures.

The effective stiffness associated with this arrangement of materials can be obtained using combinations of the Voigt and Reuss assumptions.

The central region of the composite can be treated using the isostress assumption to obtain:

$$E_{mid} = \left(\frac{\sqrt{\phi_r}}{\mathrm{E_r}} + \frac{2\delta}{\mathrm{E_m}} \right)^{-1} \tag{11.17}$$

The overall composite stiffness can then be found using the isostrain assumption:

$$E^*_{hybrid} = 2\delta * E_m + \sqrt{\phi_r} * E_{mid} \tag{11.18a}$$

$$= \left(1-\sqrt{\phi_r}\right) * E_m + \frac{\sqrt{\phi_r}}{\frac{\sqrt{\phi_r}}{\mathrm{E_r}} + \frac{\left(1-\sqrt{\phi_r}\right)}{\mathrm{E_m}}} \tag{11.18b}$$

The modulus predicted by the hybrid rule is always greater than or equal to that of the Reuss rule and less than or equal to that of the Voigt rule.

Example 11.1:

For PCC assume that the Young's modulus of the hcp is 20 GPa and that the Young's modulus of the aggregate is 100 GPa. Also, assume that the aggregate volume fraction in the PCC is 75%. Compute the different Young's modulus estimates for the PCC given based on:

a) The Voigt isostrain rule;
b) The Reuss isostress rule; and
c) The hybrid rule of mixtures.

Solution: ϕ_r=0.75; ϕ_m=0.25; E_m=20 GPa; E_r=100 GPa.

- $$E^*_{voigt} = \phi_r E_r + \phi_m E_m = 0.75*100 + 0.25*20 = 80 \text{ GPa}$$
- $$E^*_{reuss} = \left(\frac{\phi_r}{\mathrm{E_r}} + \frac{\phi_m}{\mathrm{E_m}}\right)^{-1} = \left(\frac{.75}{100} + \frac{.25}{20}\right)^{-1} = 50 \text{ GPa}$$
- $$E^*_{hybrid} = \left(1 - \sqrt{\phi_r}\right) * E_m + \frac{\sqrt{\phi_r}}{\frac{\sqrt{\phi_r}}{\mathrm{E_r}} + \frac{\left(1-\sqrt{\phi_r}\right)}{\mathrm{E_m}}} = 2.7 \text{ GPa} + 56.4 \text{GPa} = 59.1 \text{GPa}$$

Thus, the modulus from the hybrid rule is indeed intermediate to that of the Voigt and Reuss rules, as would be expected.//

D. Empirical Relations for Strength & Stiffness of PCC

1. Realistic values for the Young's modulus of stone that might be used as aggregate in pcc ranges from: 20 GPa $\leq$Er $\leq$150 GPa, with slates, shales, and sandstones having the lower stiffnesses and granites and limestone having the higher stiffnesses.

2. Mechanical Properties of Hydrated Cement Paste:

   - For hydrated cement paste, both the elastic stiffness and the shear strength tend to decrease as the capillary porosity in the hcp increases.
   - Different semi-empirical models exist that relate the elastic stiffness of hcp to the capillary porosity. One in particular [from Concrete 2nd Ed., by Mindess, Young and Darwin, Prentice-Hall, (2003)] is :

$$E_{hcp} = 29\,\text{GPa} * (1 - n_c)^3 \qquad (11.19)$$

   where n_c represents the capillary porosity of the hcp. Thus, the higher the capillary porosity is, the lower the stiffness of the hcp will be.

   - As the large capillary voids in hcp increase, the fracture stress decreases with the square root of the typical void size. (The Griffith microcrack model presented in the Period #3 notes is the basis for this statement.)

3. Empirical Formulae Relating E and f_c'

   - Although the shear strength of pcc is typically a function of the strengths of both the matrix phase (hcp) and the reinforcement phase (aggregate), the hcp is usually the weakest link that limits the strength of pcc.

• This helps to explain why there are empirical relations between the unconfined compressive strength f'_c of pcc and its Young's modulus. One such model is as follows:

$$E_{pcc} = 4730\,\text{MPa} * \sqrt{f_c'} \quad \text{where } \mathrm{f_c}' \text{ is in units of MPa} \qquad (11.20)$$

• Another empirical relation between the static Young's modulus of pcc E and f_c' is:

$$E_{pcc} = 20\,\text{GPa} + 200\,f_c' \qquad \text{where } \mathrm{f_c}' \text{ is in units of GPa} \qquad (11.21)$$

• PCC has viscoelastic elastic characteristics that make it stiffer when loaded rapidly and less stiff when loaded over a long period of time. An empirical relation similar to that of (11.21) for the *dynamic* modulus of PCC is:

$$\left(E_{pcc}\right)_{dynamic} = 31\,\text{GPa} + 160\,f_c' \qquad \text{where } \mathrm{f_c}' \text{ is in units of GPa} \qquad (11.22)$$

• The normal range for the unconfined compressive strength of pcc is roughly:

$$2000\,\text{psi} \le \mathrm{f_c}' \le 8000\text{psi} \qquad (11.23a)$$

$$14\,\text{MPa} \le \mathrm{f_c}' \le 56\text{MPa} \qquad (11.23b)$$

When the unconfined compressive strength of pcc exceeds 10,000 psi (70 MPa), the concrete is usually called *"high-performance concrete"* or hpc. Such high-performance concrete (hpc) is generally achieved by using blended cements (i.e. those in which cement replacement has been employed) together with low water-cement ratios.

The Poisson's ratio of pcc usually decreases with an increasing aggregate volume fraction:

- For pure hcp (concrete w/o any aggregate) $\nu \approx 0.20 – 0.28$;
- For aggregate volume fractions in the range of 70-80%, $\nu \approx 0.10 – 0.18$;

If one knows the Young's modulus of pcc and the Poisson's ratio, then the shear modulus can be obtained using the following formula appropriate for isotropic, linearly elastic materials"

$$G = \frac{E}{2(1+\nu)} \tag{11.24}$$

It is worth mentioning that in pcc, the matrix phase or the hcp is usually quite a bit weaker than the aggregate phase. If one considers the rule of mixtures composite models applied to pcc, both the Reuss isostress model (Fig. 11.3) and the hybrid model (Fig. 11.4) are such that the strength of pcc is controlled by the weakest constituent (or the weakest link). Thus these models predict that:

$$f'_c = f'_{hcp} \tag{11.25}$$

The Voigt or iso-strain model is not realistic for pcc and thus would over-estimate the strength of pcc.

E. Measuring the Strength of PCC

It is common to measure the unconfined compressive strength of pcc and less common, although still sometimes done, to measure the tensile strength.

1. Unconfined Compressive Strength

This test is usually performed on cylinders of 6" dia. (150mm) and 12" height (300mm). The test can also be performed on smaller cylinders and also on smaller cubes of pcc.

It is usually best to use specimens with a height to diameter ratio of 2 to 1. With this ratio, the confining effects of friction between the specimen and the loading platens is reduced. With specimens having an aspect ratio of 1 (height to width = 1), the confining effect tends to make the measured strength of the pcc higher than it would otherwise be. Therefore, the measured f_c' of cubical specimens would tend to larger than that of 2:1 cylindrical specimens.

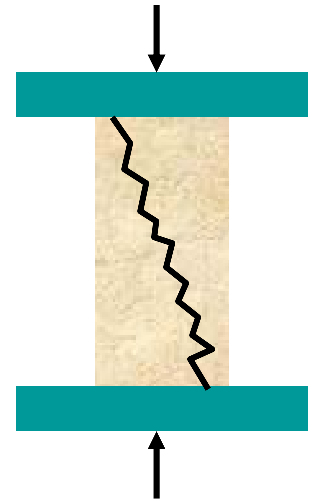

Fig. 11.5. Schematic of unconfined compression test.

2. Tensile Strength Testing

There are two common procedures for measuring the tensile strength of pcc. These are: (a) the split cylinder test; and (b) the bending test.

a) **split cylinder test.** This test is performed by taking a 2:1 pcc cylinder, turning it on its side (Fig. 11.6) and then loading it to failure. The tensile strength of the pcc from this test is given as follows:

$$f'_t = \frac{2P}{\pi d h} \qquad (11.26)$$

where *d* is the diameter of the cylinder, *h* is its height, and *P* is the magnitude of the load at failure. It is also worth mentioning here that the load P is applied over the full length of the cylinder edge rather than just at a point.

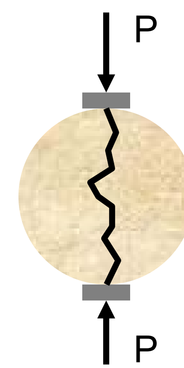

Fig. 11.6. Schematic of the split cylinder test.

b) The bending test: In this test, a prismatic beam like that of Fig. 11.7 is loaded at the one-third points. When the beam (of depth h, width b, and length L) ruptures , the peak tensile bending stress in the middle third of the beam is the tensile strength of the pcc:

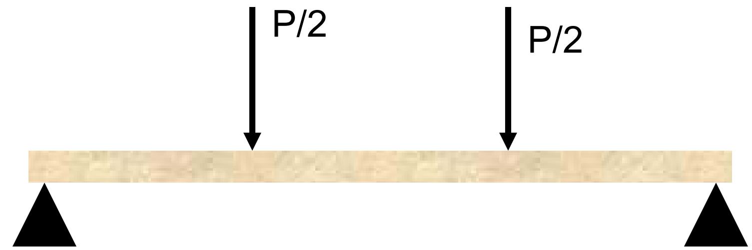

Fig. 11.7. Bending test to measure f_t'.

$$f_t' = \sigma_t^{\max} = \frac{M_{\max} * c}{I} = \frac{\left(\frac{PL}{6}\right) * \frac{h}{2}}{\frac{bh^3}{12}} = \frac{PL}{bh^2} \qquad (11.27)$$