53:139 Foundation Engineering

Homework #3 Solutions

Spring Semester, 2009

Problem 2.1.

For a Shelby tube, given: outside diameter = 2 in. and inside diameter 1 7/8 in.

a. What is the area ratio of the tube?
b. The outside diameter remaining the same, what should be the inside diameter of the tube to give an area ratio of 10%.

a. Eq. (2.5): $A_R(\%) = \frac{D_o^2 - D_i^2}{D_i^2} \times 100 = \left[\frac{(2)^2 - (1.875)^2}{(1.875)^2}\right](100) = \mathbf{13.78\%}$

b. $A_R = \frac{10}{100} = \frac{(2)^2 - D_i^2}{D_i^2} = \mathbf{1.907\ in.}$

Problem 2.2.

The soil profile shown in Figure P2.2 along with the standard penetration numbers in the clay layer, use Eqs. (2.11) and (2.12) to determine and plot the variation of c_u and OCR with depth.

Depth from ground surface (m)	N_{60}	c_u [a] (kN / m²)	σ'_o (MN / m²)	OCR [b]
4.0	6	**105.4**	$\frac{1}{1000}[2(17) + 2(19 - 9.81)] = 0.0524$	**5.06**
5.5	9	**141.1**	$0.0524 + \frac{1}{1000}(17 - 9.81)(1.5) = 0.0632$	**5.88**
7.0	8	**129.6**	$0.0632 + \frac{1}{1000}(17 - 9.81)(1.5) = 0.074$	**4.86**
8.5	10	**152.2**	$0.074 + \frac{1}{1000}(17 - 9.81)(1.5) = 0.0848$	**5.16**
10.0	11	**163.0**	$0.0848 + \frac{1}{1000}(17 - 9.81)(1.5) = 0.0956$	**5.08**

[a] $c_u\ (\text{kN/m}^2) = 29N_{60}^{0.72}$; [b] $\text{OCR} = 0.193(N_{60}/\sigma'_o)^{0.689}$

Problem 2.3.

Following is the variation of the field standard penetration number (N_{60}) in a sand deposit. The groundwater table is located at a depth of 5.5m. Given the dry unit weight of sand from 0 to a depth of 5.5m is 18.08 kN/m^3. Use the relationship of Liao and Whitman given in Eq. (2.14) to calculate the corrected penetration numbers.

Depth (m)	σ'_o (kN / m^2)
1.5	$18.08 \times 1.5 = 27.12$
3	$18.08 \times 3.0 = 54.24$
4.5	$18.08 \times 4.5 = 81.36$
6	$18.08 \times 5.5 + (19.34 - 9.81)(0.5) = 104.2$
7.5	$18.08 \times 5.5 + (19.34 - 9.81)(2) = 118.5$
9	$18.08 \times 5.5 + (19.34 - 9.81)(3.5) = 132.8$

Eq. (2.14): $C_N = \left[\dfrac{1}{(\sigma'_o/p_a)}\right]^{0.5}$. Now the following table can be prepared.

Depth (m)	N_{60}	σ'_o (kN/m^2)	C_N	$(N_1)_{60}$ [a]
1.5	5	27.12	1.92	**10**
3	7	54.24	1.36	**10**
4.5	9	81.36	1.11	**10**
6	8	104.2	0.98	**8**
7.5	13 ~~12~~	118.5	0.92	**11**
9	12 ~~11~~	132.8	0.87	**10**

[a] rounded to nearest whole number

Problem 2.4.

For the soil profile described in Problem 2.3, estimate an average peak soil friction angle using Eqs. (2.24) and (2.25).

$$\phi' = 27.1 + 0.30 N_{60} - .00054 (N_{60})^2 \qquad (2.24)$$

$$\phi' = \tan^{-1}\left[\left(\frac{N_{60}}{12.2 + 20.3\left(\frac{\sigma_o'}{p_a}\right)}\right)^{.39}\right] \qquad (2.25)$$

Depth(m)	N_{60}	$\sigma_o'(kPa)$	ϕ' Eq. 2.24	ϕ' Eq. 2.25
1.5	5	27.12	28.6	33
3	7	54.24	29.2	33.6
4.5	9	81.36	29.8	34
6	8	104.2	29.5	31.6
7.5	13	118.5	30.9	35.2
9	12	132.8	30.6	33.8
			Average	**Average**
			29.8	**33.5**

Problem 2.7.

For the information provided, and assuming the uniformity coefficient Cu of the sand to be 3.2 and an over-consolidation ratio OCR of 3, estimate the average relative density of the sand between the depth of 10 and 20 feet. Use Eq. 2.18b of the textbook.

Eq. (2.18b): $$D_r\,(\%) = 12.2 + 0.75\left[222N_{60} + 2311 - 771\text{OCR} - 779\left(\frac{\sigma'_o}{p_a}\right) - 50C_u^2\right]^{0.5}$$

At 10 ft: $$D_r = 12.2 + 0.75\begin{bmatrix}(222)(9) + 2311 - 711(3.0) \\ -779\left(\dfrac{1150}{2000}\right) - (50)(3.2)^2\end{bmatrix}^{0.5} = 38.35\%$$

At 15 ft: $$D_r = 12.2 + 0.75\begin{bmatrix}(222)(11) + 2311 - 711(3.0) \\ -779\left(\dfrac{1725}{2000}\right) - (50)(3.2)^2\end{bmatrix}^{0.5} = 40.6\%$$

At 20 ft: $$D_r = 12.2 + 0.75\begin{bmatrix}(222)(12) + 2311 - 711(3.0) \\ -779\left(\dfrac{2030}{2000}\right) - (50)(3.2)^2\end{bmatrix}^{0.5} = 41.63\%$$

Average D_r = (⅓)(38.35 + 40.6 + 41.63) ≈ **40%**

Problem 2.8.
Referring to Fig. P2.2, vane shear tests were conducted in the clay layer. The vane dimensions were 63.5mm (D) x 127mm (H). For the test at A, the torque required to cause failure was 0.072 kN-m. For the clay, the liquid limit (LL) is 51, and the plastic limit (PL) is 18. Estimate the undrained cohesion of the clay for use in the design by using Bjerrum's λ relationship [Eq. 2.35].

Eq. (2.29): $c_u = \dfrac{T}{K}$

Eq. (2.31): $K = 366 \times 10^{-8} D^3 = 366 \times 10^{-8}(6.35)^3 = 93.7 \times 10^{-5}$

$$c_{u\,(\text{VST})} = \frac{0.072}{93.7 \times 10^{-5}} = 76.84 \text{ kN / m}^2$$

$$c_{u\,(\text{corrected})} = \lambda c_{u\,(\text{VST})} = [1.7 - 0.54\log(\text{PI})](76.84)$$

$$= [1.7 - 0.54\log(51 - 18)](76.84) = \mathbf{67.62 \text{ kN / m}^2}$$

Problem 2.10.
Referring to Problem 2.8, determine the over-consolidation ratio (OCR) for the clay at A. Use Eqs. (2.37) and (2.38).

$$\text{OCR} = \beta \frac{c_{u(\text{field})}}{\sigma'_o}; \;\; \beta = 22(\text{PI})^{-0.48} = 22(51 - 18)^{-0.48} = 4.11$$

$$\sigma'_o = (2)(17) + (2)(19 - 9.81) + (3)(17 - 9.81) = 73.95 \text{ kN / m}^2$$

$$c_{u\,(\text{VST})} = 76.84 \text{ kN/m}^2$$

$$\text{OCR} = (4.11)\left(\frac{76.84}{73.95}\right) = \mathbf{4.27}$$

Problem 2.17.
During a field exploration, coring of rock was required. The core barrel was advanced 5 ft during the coring. The length of core recovered was 3.2 ft. What was the recovery ratio?

Eq (2.70): Recovery ratio = 3.2ft/5 ft = **64%**

Problem 2.18.
The P-wave speed in a soil is 1900 m/sec. Assuming a Poisson's ratio ν of 0.32, calculate the modulus of elasticity (i.e. Young's modulus E) of the soil. Assume the unit weight γ of the soil is 18 kN/m^3.

$$v_p = \left(\frac{K + \frac{4}{3}\mu}{\rho} \right)^{\frac{1}{2}} \Rightarrow \rho * (v_p)^2 = K + \frac{4}{3}\mu$$

$$\rho = \frac{\gamma}{g} = \frac{18 kN \cdot m^{-3}}{9.81 m \cdot s^{-2}} = 1835 kg \cdot m^{-3}$$

$$K + \frac{4}{3}\mu = \rho * (v_p)^2 = 1835 kg \cdot m^{-3} (1900 m \cdot s^{-1})^2$$
$$= 6.62 \text{ GPa}$$

From linear, isotropic elasticity theory: (see for example http://en.wikipedia.org/wiki/Elastic_modulus)

$$K = \frac{E}{3(1-2\nu)}$$

$$\mu = \frac{E}{2(1+\nu)}$$

$$K + \frac{4}{3}\mu = \frac{E}{3(1-2\nu)} + \frac{4E}{6(1+\nu)} = 6.62 GPa$$

Now, using that ν=0.32, the above reduces to

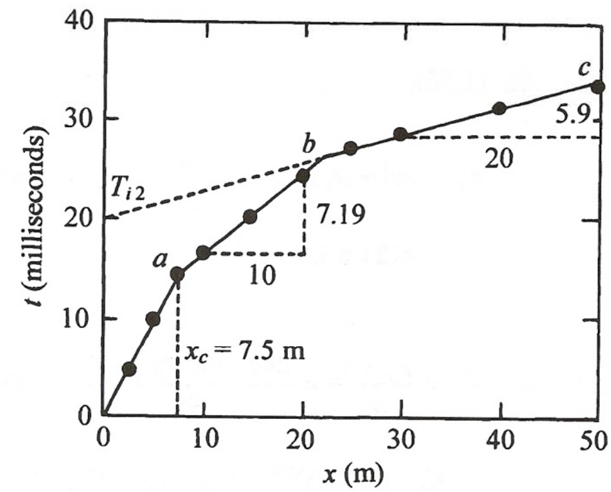
$$K + \frac{4}{3}\mu = \frac{E}{3(1-0.64)} + \frac{4E}{6(1+.32)} = 6.62 GPa$$

$$1.43E \quad = 6.62 GPa$$

$$E \quad = 4.63 \, GPa$$

Problem 2.19.
The results of a seismic refraction survey are provided. Determine the thickness and the p-wave speeds of the materials encountered.

A time-distance plot is shown.



$$\text{Slope of } 0a = \frac{1}{v_1} = \frac{15.24\times10^{-3}}{7.5}$$

$$v_1 = \frac{7.5\times10^{-3}}{15.24} = \textbf{492 m / sec (top layer)}$$

$$v_2 = \frac{10\times10^{3}}{7.19} \approx \textbf{1390 m / sec}$$

$$v_3 = \frac{20\times10^{3}}{5.19} \approx \textbf{3390 m / sec}$$

$$x_c = 7.5\text{m}$$

$$Z_1 = \frac{1}{2}\sqrt{\frac{v_2 - v_1}{v_2 + v_1}}x_c = \frac{1}{2}\sqrt{\frac{1390-492}{1390+492}} \times 7.5 = \textbf{2.6 m}$$

$$\text{Eq. (2.74):} \quad Z_2 = \frac{1}{2}\left[T_{i2} - \frac{2Z_1\sqrt{v_3^2 - v_1^2}}{(v_3)(v_1)}\right]\left[\frac{v_3 v_2}{\sqrt{v_3^2 - v_2^2}}\right]; \; T_{i2} \approx 20\times10^{-3} \text{ sec}$$

$$\frac{2Z_1\sqrt{v_3^2 - v_1^2}}{(v_3)(v_1)} = \frac{(2)(2.6)\sqrt{(3390)^2 - (492)^2}}{(3390)(492)} = 0.0105$$

$$\frac{(v_3)(v_2)}{\sqrt{v_3^2 - v_2^2}} = \frac{(3390)(1390)}{\sqrt{(3390)^2 - (1390)^2}} = 1524$$

So, $Z_2 = (½)(0.02 - 0.0105)(1524) = \textbf{7.24 m}$