53:139 Foundation Engineering

Homework #4 Solutions

Spring Semester 2009

CHAPTER 3

3.1 a. Eq. (3.3) and Table 3.1: $q_{\text{all}} = \frac{q_u}{\text{FS}} = \frac{1}{\text{FS}}\left(c'N_c + qN_q + \frac{1}{2}\gamma BN_\gamma\right)$

For $\phi' = 28°$, $N_c = 31.61$, $N_q = 17.81$, $N_\gamma = 13.7$

$$q_{\text{all}} = \frac{1}{4}\left[(400)(31.61) + (110)(3)(17.81) + \frac{1}{2}(110)(3)(13.7)\right] = \mathbf{5195\ lb/ft^2}$$

b. $q_{\text{all}} = \frac{q_u}{\text{FS}} = \frac{1}{\text{FS}}\left(c'N_c + qN_q + \frac{1}{2}\gamma BN_\gamma\right)$

For $\phi' = 35°$, from Table 3.1, $N_c = 57.75$, $N_q = 41.44$, $N_\gamma = 45.41$

$$q_{\text{all}} = \frac{1}{4}\left[0 + (1.2 \times 17.8)(41.44) + \frac{1}{2}(17.8)(1.5)(45.41)\right] = \mathbf{372.8\ kN/m^2}$$

c. Table 3.1: $\phi' = 30°$, $N_q = 22.46$, $N_\gamma = 19.13$

Eq. (3.7), with $c' = 0$

$$q_{\text{all}} = \frac{q_u}{\text{FS}} = \frac{1}{\text{FS}}(qN_q + 0.4\gamma BN_\gamma)$$

$$= \frac{1}{4}[(2 \times 16.5)(22.46) + (0.4)(16.5)(3)(19.13)] = \mathbf{280\ kN/m^2}$$

3.3 a. In Eq. (3.25), all inclination factors are unity. Also, since it is a strip foundation, B/L = 0. So all shape factors are equal to unity. Therefore

$$q_u = c'N_cF_{cd} + qN_qN_{qd} + \frac{1}{2}\gamma BN_\gamma F_{\gamma d}$$

Table 3.3: $\phi' = 28°$, $N_c = 25.8$, $N_q = 14.72$, $N_\gamma = 16.72$

Eq. (3.30): $F_{cd} = 1 + 0.4(D_f/B) = 1 + 0.4(3/3) = 1.4$

Eq. (3.31): $F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2(D_f/B) = 1 + (0.299)(3/3) = 1.299$

Eq. (3.32): $F_{\gamma d} = 1$

$$q_u = (400)(25.8)(1.4) + (3)(110)(14.72)(1.299) + \frac{1}{2}(110)(3)(16.72)(1)$$

$$= 14{,}448 + 6310 + 2758.8 \approx 23{,}517 \text{ lb / ft}^2$$

$q_{all} = q_u/4 \approx$ **5879 lb / ft²**

b. $F_{ci} = F_{qi} = F_{\gamma i} = 1$; $F_{cs} = F_{qs} = F_{\gamma s} = 1$; $F_{\gamma d} = 1$; $c' = 0$

$$q_{all} = \frac{q_u}{\text{FS}} = \frac{1}{\text{FS}}(qN_q + \frac{1}{2}\gamma BN_\gamma)$$

Table 3.3: $N_q = 33.3$; $N_\gamma = 48.03$

$F_{qd} = 1 + 2\tan\phi'(1\text{-}\sin\phi')^2(D_f/B) = 1 + 0.25(1.2/1.5) = 1.2$

$q_{all} = \frac{1}{4}[(1.2)(17.8)(33.3)(1.2) + \frac{1}{2}(17.8)(1.5)(48.03)] =$ **373.7 kN / m²**

c. $q_u = qN_qF_{qd}F_{qs} + \frac{1}{2}\gamma BN_\gamma F_{\gamma d}F_{\gamma s}$

$\phi' = 30°$. From Table 3.3, $N_q = 18.4$; $N_\gamma = 22.4$

Eqs. (3.29) and (3.32): $F_{\gamma s} = 0.6$; $F_{\gamma d} = 1$

Eq. (3.28): $F_{qs} = 1 + \tan\phi' = 1.577$

Eq. (3.31): $F_{qd} = 1 + 2\tan\phi'(1\text{-}\sin\phi')^2(D_f/B) = 1 + 0.29(2/3) = 2.294$

$q_{all} = \frac{1}{4}[(2)(16.5)(18.4)(1.193)(1.577) + \frac{1}{2}(16.5)(3)(22.4)(0.6)(1) =$ **368.8 kN / m²**

3.4 $$Q_{all} = \left(\frac{1}{\text{FS}}\right)B^2\left[c'N_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + \tfrac{1}{2}\gamma BN_\gamma N_{\gamma s}N_{\gamma d}N_{\gamma i}\right]$$

$\phi' = 25°$; Table 3.3: $N_c = 20.72$, $N_q = 10.66$, $N_\gamma = 10.88$

$$F_{cs} = 1 + \frac{B}{L}\frac{N_q}{N_c} = 1 + \left(\frac{5.5}{5.5}\right)\left(\frac{10.66}{20.72}\right) = 1.514$$

$$F_{qs} = 1 + \frac{B}{L}\tan\phi' = 1 + \left(\frac{5.5}{5.5}\right)\tan 25 = 1.466$$

$$F_{\gamma s} = 1 - 0.4\frac{B}{L} = 1 - 0.4(1) = 0.6$$

$$F_{cd} = 1 + 0.4\left(\frac{D_f}{B}\right) = 1 + 0.4\left(\frac{4}{5.5}\right) = 1.29$$

$$F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2\frac{D_f}{B} = 1 + 0.311\left(\frac{4}{5.5}\right) = 1.226$$

$$F_{\gamma d} = 1$$

$$F_{ci} = F_{qi} = \left(1 - \frac{15}{90}\right)^2 = 0.694$$

$$F_{\gamma i} = \left(1 - \frac{\beta}{\phi'}\right)^2 = \left(1 - \frac{15}{25}\right)^2 = 0.16$$

$$q_{\text{all}} = \left(\frac{1}{4}\right)\left(\frac{5.5^2}{1000}\right)\begin{bmatrix}(350)(20.72)(1.514)(1.29)(0.694)\\ +(107\times 4)(10.66)(1.466)(1.226)(0.694)\\ +(0.5)(107)(5.5)(10.88)(0.6)(1)(0.16)\end{bmatrix} = \mathbf{119.7\ kip}$$

3.5 $$Q_{all} = \left(\frac{1}{\text{FS}}\right) B \times L\left[c'N_c F_{cs} F_{cd} F_{ci} + qN_q F_{qs} F_{qd} F_{qi} + \tfrac{1}{2}\gamma B N_\gamma N_{\gamma s} N_{\gamma d} N_{\gamma i} - q'\right]$$

$$q' = (1)(16.8) + (1)(19.4 - 9.81) = 26.39 \text{ kN / m}^2$$

$\phi' = 25°$. Table 3.3: $N_c = 20.72$; $N_q = 10.66$; $N_\gamma = 10.88$

$$F_{cs} = 1 + \left(\frac{2}{3}\right)\left(\frac{10.66}{20.72}\right) = 1.343$$

$$F_{qs} = 1 + \left(\frac{2}{3}\right)\tan 25 = 1.31$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{2}{3}\right) = 0.73$$

$$F_{cd} = 1 + 0.4\left(\frac{2}{2}\right) = 1.4$$

$$F_{qd} = 1 + 0.311\left(\frac{2}{2}\right) = 1.311$$

$$F_{\gamma x} = 1$$

$$Q_{\text{all}} = \left(\frac{1}{4}\right)(2 \times 3)\begin{bmatrix}(50)(20.72)(1.343)(1.4) + (26.39)(10.66)(1.31)(1.311) \\ +(0.5)(19.4 - 9.81)(2)(10.88)(0.73)(1) - 26.39\end{bmatrix} = \mathbf{3721\ kN}$$

3.6 $Q_{all} = 150{,}000\ \text{lb} = 150\ \text{kip}$; $Q_u = (FS)(Q_{all}) = (3)(150) = 450\ \text{kip}$

Also, $q = \gamma D_f = (0.115)(3) = 0.345\ \text{kip / ft}^2$

$$q_u = \frac{450}{B^2} \qquad \text{(a)}$$

For $\phi' = 40°$, Table 3.3 gives $N_q = 64.20$, $N_\gamma = 109.41$

$F_{qs} = 1 + (B/L)\tan\phi' = 1 + (B/B)\tan 40° = 1.839$; $F_{qd} = 1 + 0.214(3/B)$

$F_{\gamma s} = 1 - 0.4(B/B) = 0.6$; $F_{\gamma d} = 1$

$$q_u = (0.345)(64.20)(1.839)\left(1 + \frac{0.642}{B}\right) + \frac{1}{2}(0.115)(B)(109.41)(0.6)(1)$$

$$= 40.73 + \frac{26.15}{B} + 3.77B \qquad \text{(b)}$$

Combining Eqs. (a) and (b)

$$\frac{450}{B^2} = 40.73 + \frac{26.15}{B} + 3.77B$$

By trial and error, Eq. (c) gives $B \approx$ **2.75 ft**

3.9 $B' = B - 2e = 1.5 - 2(0.15) = 1.2$ m; $L = 1.5$ m

$q_u = q N_q F_{qs} F_{qd} + \frac{1}{2} \gamma B' N_\gamma F_{\gamma s} F_{\gamma d}$

Table 3.3: $\phi' = 36°$; $N_q = 37.75$; $N_\gamma = 56.31$

$F_{qs} = 1 + (B'/L)\tan\phi' = 1 + (1.2/1.5)\tan 36 = 1.58$

$F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2 (D_f/B) = 1 + 2\tan 36(1 - \sin 36)^2(1/1.5) = 1.165$

$F_{\gamma s} = 1 - 0.4(B/L) = 1 - 0.4(1.2/1.5) = 0.68$

$F_{\gamma d} = 1$

$q_u = (1 \times 17)(37.75)(1.58)(1.165) + (\frac{1}{2})(17)(1.2)(56.31)(0.68)(1)$

$= 1571.9 \text{ kN / m}^2$

$$Q_{all} = \frac{q_u B' L}{FS} = \frac{(1571.9)(1.2)(1.5)}{4} = \mathbf{707.3\ kN}$$

3.13 $e_B/B = 0.4/4 = 0.1$; $e_L/L = 1.2/6 = 0.2$. So Case II, Figure 3.16 applies.

From Figure 3.16, $L_1/L = 0.865$ and $L_2/L = 0.22$

$L_1 = (0.865)(6) = 5.19$ ft; $L_2 = (0.22)(6) = 1.32$ ft

Eq. (3.64): $A' = \frac{1}{2}(L_1 + L_2)B = \frac{1}{2}(5.19 + 1.32)(4) = 13.02 \text{ ft}^2$

Eq. (3.65): $B' = A'/L_1 = 13.02/5.19 = 2.51$ ft

Eq. (3.66): $L' = 5.19$ ft

$q'_u = qN_q F_{qs} F_{qd} + \frac{1}{2}\gamma' B' N_\gamma F_{\gamma s} F_{\gamma d}$

Table 3.3: $\phi' = 35°$; $N_q = 33.3$; $N_\gamma = 48.03$

$F_{qs} = 1 + (B'/L')\tan\phi' = 1 + (2.51/5.19)\tan 35 = 1.339$

$F_{\gamma s} = 1 - 0.4(B'/L') = 1 - 0.4\,(2.51/5.19) = 0.806$

$F_{qd} = 1 + 2\tan 35(1 - \sin 35)^2(3/4) = 1.191$

$F_{\gamma d} = 1$

$q'_u = (115 \times 3)(33.3)(1.339)(1.191) + \frac{1}{2}(115)(2.51)(48.03)(0.806)(1) = 23{,}908 \text{ lb / ft}^2$

$$Q_{\text{all}} = \frac{q_u B'L'}{\text{FS}} = \frac{(23{,}908)(2.51)(5.19)}{(4)(1000)} = \mathbf{77.86\ kip}$$

3.14 $e_B/B = 1.5/4 = 0.375$; $e_L/L = 0.06/6 = 0.01$. So Case III, Figure 3.17 applies.

From Figure 3.17, $B_1/B = 0.3$ and $B_2/B = 0.25$

$B_1 = (0.3)(4) = 1.2$ ft; $B_2 = (0.25)(4) = 1$ ft

Eq. (3.67): $A' = \frac{1}{2}(B_1 + B_2)L = \frac{1}{2}(1.2 + 1)(6) = 6.6 \text{ ft}^2$

Eq. (3.68): $B' = A'/L = 6.6/6 = 1.1$ ft

Eq. (3.66): $L' = L = 6$ ft

$F_{qs} = 1 + (B'/L')\tan\phi' = 1 + (1.1/6)\tan 35 = 1.128$

$F_{\gamma s} = 1 - 0.4(B'/L') = 1 - 0.4\,(1.1/6) = 0.927$

$F_{qd} = 1 + 2\tan 35(1 - \sin 35)^2(3/4) = 1.191$

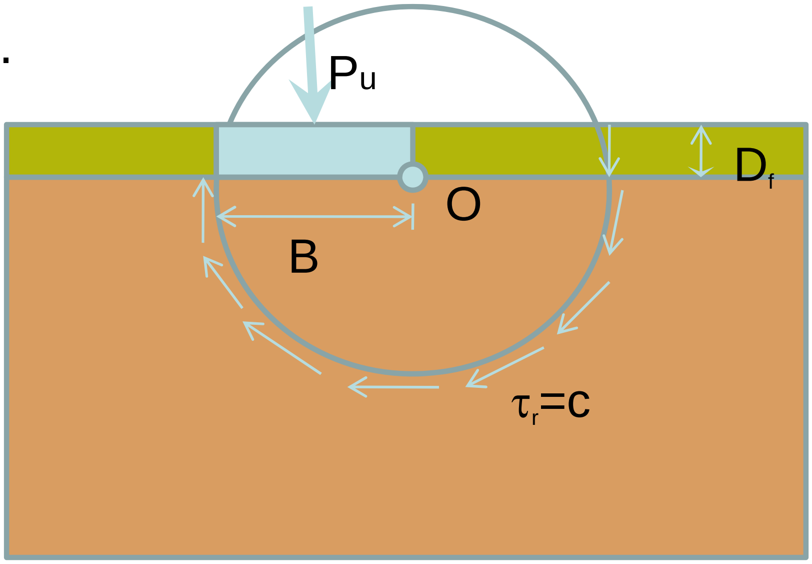
$F_{\gamma d} = 1$

$q'_u = (115 \times 3)(33.3)(1.128)(1.191) + \frac{1}{2}(115)(1.1)(48.03)(0.927)(1) = 18{,}250 \text{ lb/ft}^2$

$$Q_{\text{all}} = \frac{q_u B'L'}{\text{FS}} = \frac{(18{,}250)(1.1)(6)}{(4)(1000)} = \mathbf{30.1\ kip}$$

SUPPLEMENTAL PROBLEMS

#1.



The failure mechanism in this case involves rotation about point O. The force P_u is the bearing capacity and drives the failure. Three mechanisms resist failure: (1) the surcharge loading to the right of the foundation; (2) the cohesion stress in the surcharge layer; and (3) the cohesion stress along the circular surface of the mechanism. These can each be quantified one at a time.

(1) The surcharge loading: Taking moments about point O, we get that:

$$\sum_O M = 0 = (P_u)_q * \frac{B}{2} - (\gamma D_f B) * \frac{B}{2} \Rightarrow (P_u)_q = \gamma D_f B$$

(2) The cohesion of the surcharge layer gives additional resistance:

$$\sum_O M = 0 = (P_u)_{cD} * \frac{B}{2} - (cD_f) * B \Rightarrow (P_u)_{cD} = 2cD_f$$

(3) The cohesive stress acting over the circular arc of radius B gives additional resistance. Once again, taking moments about point O, we get that:

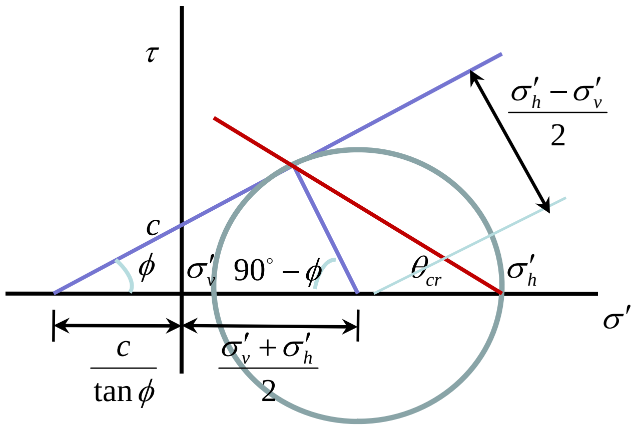
$$\sum_O M = 0 = (P_u)_{cB} * \frac{B}{2} - \int_0^{\pi} (cBd\theta)B \Rightarrow (P_u)_{cB} = 2\pi Bc$$

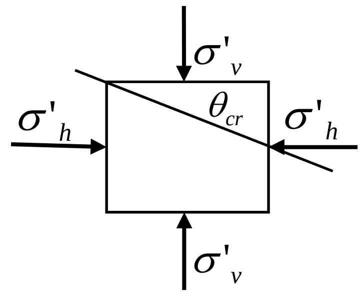
Summing the three resistance capacities gives the ultimate capacity associated with the mechanism:

$$\boxed{P_u = (P_u)_q + (P_u)_{cD} + (P_u)_{cB} = \gamma D_f B + 2cD_f + 2\pi cB}$$

PROBLEM #2:

When the vertical stress in a soil is known and the horizontal stress is increased until shear failure occurs, the Mohr's circle of the stress state at failure will be as shown:





From the right triangle:

$$\sin\phi = \frac{\frac{1}{2}\left(\sigma'_h - \sigma'_v\right)}{\frac{c}{\tan\phi} + \frac{\sigma'_h + \sigma'}{2}}$$

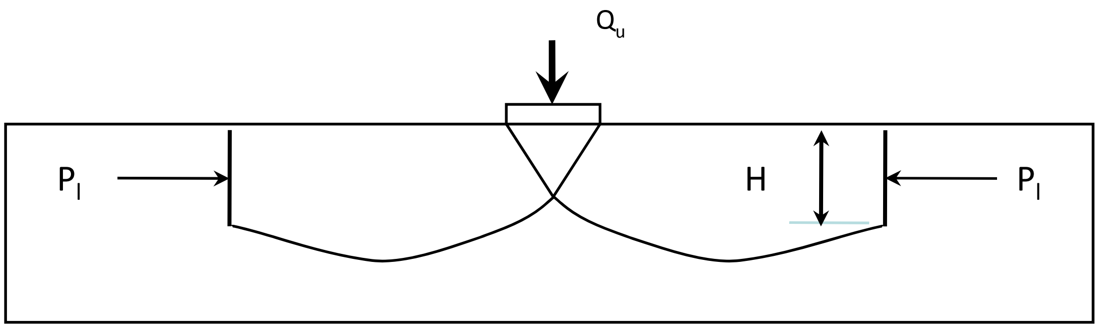
From circle geometry

$$\theta_{cr} = \tfrac{1}{2}\left(90^\circ - \phi\right) = 45^\circ - \frac{\phi}{2}$$

After algebraic manipulation it can be shown that

$$\sigma'_h = K_p \sigma'_v + 2c\sqrt{K_p} \quad \text{where } K_p = \frac{1+\sin\phi}{1-\sin\phi} = \tan^2\left(45^\circ + \frac{\phi}{2}\right)$$

This result was used in developing Terzaghi's bearing capacity model for strip footings in which the assumed failure mechanism in the soil was as shown below. In order for sliding to occur along the logarithmic spirals, the lateral forces P_l in the Rankine passive wedges has to be overcome.



The three mechanisms that contribute to the ultimate bearing capacity Q_u are: (1) the cohesion of the soil; (2) the surcharge loading; and (3) the unit weight of the soil and the friction. Each of the three mechanisms lead to a different vertical stress in the soil, and thus a different horizontal passive stress along acting on the shaded triangular wedges that generates shear failure. Specifically:

$$q = 0;\ \gamma = 0;\ \mathrm{c} > 0;$$
$$\sigma'_v = 0 \Rightarrow \sigma'_h = 2c\sqrt{K_p}$$
$$\mathrm{P_l} = \mathrm{H} * \sigma'_h = 2cH\sqrt{K_p}$$

$$q > 0;\ \gamma = 0;\ \mathrm{c} = 0;$$
$$\sigma'_v = q \Rightarrow \sigma'_h = qK_p$$
$$P_l = H * qK_p$$

$$q = 0;\ \gamma > 0;\ \mathrm{c} = 0;$$
$$\sigma'_v = \gamma z \Rightarrow \sigma'_h = \gamma z K_p$$
$$P_l = \frac{\gamma H^2 K_p}{2}$$

For each of these three separate cases, the lateral force P_l is related back to the ultimate bearing load Q_u using principles of statics.