53:139 Foundation Engineering

Homework #5 Solutions

Spring Semester, 2009

5.6 The plan of the foundation can be divided into 4 areas, each measuring 2.5 ft × 2.5 ft.

$H_1 = 3$ ft; $H_2 = 13$ ft; $B = 2.5$ ft; $L = 2.5$ ft

$$m_2 = \frac{B}{H_1} = \frac{2.5}{3} = 0.833; \quad n_2 = \frac{L}{H_1} = \frac{2.5}{3} = 0.833$$

Figure 5.7: $I_{a(H_1)} = 0.212$

$$m_2 = \frac{B}{H_2} = \frac{2.5}{13} = 0.192 \; (\text{~~0.833~~}); \quad n_2 = \frac{L}{H_2} = \frac{2.5}{13} = 0.192$$

Figure 5.7: $I_{a(H_2)} =$ ~~0.13~~ 0.088

$$\Delta\sigma_{\mathrm{av}(H_1/H_2)} = \left[\frac{H_2 I_{a(H_2)} - H_1 I_{a(H_1)}}{H_2 - H_1}\right](q_o)(4)$$

$$= \left[\frac{(13)(0.088) - (3)(0.212)}{13-3}\right]\left(\frac{50 \times 2000}{5 \times 5}\right)(4) = 812.8$$

~~1686 lb/ft²~~

5.7 $$\Delta\sigma_t = \frac{50 \times 2000}{(5+3)^2} = 1562.5 \text{ lb/ft}^2$$

$$\Delta\sigma_m = \frac{50 \times 2000}{(5+8)^2} = 591.7 \text{ lb/ft}^2$$

$$\Delta\sigma_b = \frac{50 \times 2000}{(5+13)^2} = 308.6 \text{ lb/ft}^2$$

$$\Delta\sigma_{\mathrm{av}} = \frac{1}{6}\,[1562.5 + (4)(591.7) + 308.6] = \mathbf{706.3\ lb/ft^2}$$

5.9 Eq. (5.25): $S_e = q_o(\alpha B')\frac{1-\mu_s^2}{E_s} I_s I_f$

$$B' = \frac{B}{2} = \frac{3}{2} = 1.5 \text{ m}$$

$q_o = 180 \text{ kN/m}^2$; $\mu_s = 0.3$; $\alpha = 4$

$$m' = \frac{L}{B} = \frac{4.6}{3} = 1.53$$

$$n' = \frac{H}{\left(\frac{B}{2}\right)} = \frac{\infty}{\left(\frac{B}{2}\right)} = \infty$$

Eq. (5.26) and Tables 5.4 and 5.5:

$$I_s = F_1 + \frac{1-2\mu_s}{1-\mu_s} F_2 = 0.669 + \frac{1-2\mu_s}{1-\mu_s}(0) = 0.669$$

$$\frac{D_f}{B} = \frac{2}{3} = 0.67; \ \frac{L}{B} = 1.53$$

Figure 5.15b: $I_f = 0.725$

$$S_e = (180)(4 \times 1.5)\left(\frac{1-0.3^2}{8500}\right)(0.669)(0.725) = 0.056 \text{ m} = \mathbf{56\ mm}$$

5.10 $m' = \frac{L}{B} = \frac{4.6}{3} = 1.53$

$$n' = \frac{H}{\left(\frac{B}{2}\right)} = \frac{4}{\left(\frac{3}{2}\right)} = 2$$

Tables 5.4 and 5.5: $F_1 = 0.292; F_2 = 0.085$

$$I_s = F_1 + \frac{1-2\mu_s}{1-\mu_s}F_2 = 0.292 + \frac{1-(2)(0.3)}{1-0.3}(0.085) = 0.34$$

$$\frac{D_f}{B} = \frac{5}{3} = 1.67$$

Figure 5.15b: $I_f = 0.62$

$$S_e = q_o(\alpha B')\frac{1-\mu_s^2}{1-\mu_s}I_s I_f = (180)(4\times 1.5)\left(\frac{1-0.3^2}{8500}\right)(0.34)(0.62)$$

$= 0.024$ m = **24.4 mm**

5.11 Eqs. (5.25) and (5.33): $S_e = 0.93q_o(\alpha B')\frac{1-\mu_s^2}{E_s}I_s I_f$

$$B' = \frac{6.25}{2} = 3.125 \text{ ft}$$

$$m' = \frac{L}{B} = \frac{10}{6.25} = 1.6$$

$$n' = \frac{H}{\left(\frac{B}{2}\right)} = \frac{32}{\left(\frac{6.25}{2}\right)} = 10.24$$

Tables 5.4 and 5.5: $F_1 = 0.597; F_2 = 0.025$

$$I_s = F_1 + \frac{1-2\mu_s}{1-\mu_s}F_2 = 0.597 + \frac{1-(2)(0.3)}{1-0.3}(0.025) = 0.611$$

$$D_f = 2.5 \text{ ft}; \quad \frac{D_f}{B} = \frac{2.5}{6.25} = 0.4$$

Figure 5.15b: $I_f = 0.83$

$$S_e = (0.93)(3000)\left(4 \times \frac{6.25}{2}\right)\frac{1-0.3^2}{3200 \times 144}(0.611)(0.83) = 0.0349 \text{ ft} = \mathbf{0.419 \text{ in.}}$$

5.15 Eq. (5.39): $S_e = \frac{q_o B_e I_G I_F I_E}{E_o}(1-\mu_s^2)$

$q_o = 150 \text{ kN/m}^2$

$$B_e = \sqrt{\frac{4B^2}{\pi}} = \sqrt{\frac{(4)(3)^2}{\pi}} = 3.385 \text{ m}$$

$\mu_s = 0.3$; $E_o = 16{,}000 \text{ kN/m}^2$

$$\beta = \frac{E_o}{kB_e} = \frac{16{,}000}{(400)(3.385)} = 11.82$$

$$\frac{H}{B_e} = \frac{20}{3.385} = 5.91$$

From Figure 5.19, $I_G \approx 0.89$. From Eq. (5.40):

$$I_F = \frac{\pi}{4} + \frac{1}{4.6 + 10\left(\dfrac{E_f}{E_o + \dfrac{B_e}{2}k}\right)\left(\dfrac{2t}{B_e}\right)^3}$$

$$= \frac{\pi}{4} + \frac{1}{4.6 + 10\left[\dfrac{15\times10^6}{16{,}000 + \left(\dfrac{3.385}{2}\right)(400)}\right]\left[\dfrac{(2)(0.25)}{3.385}\right]^3} = 0.815$$

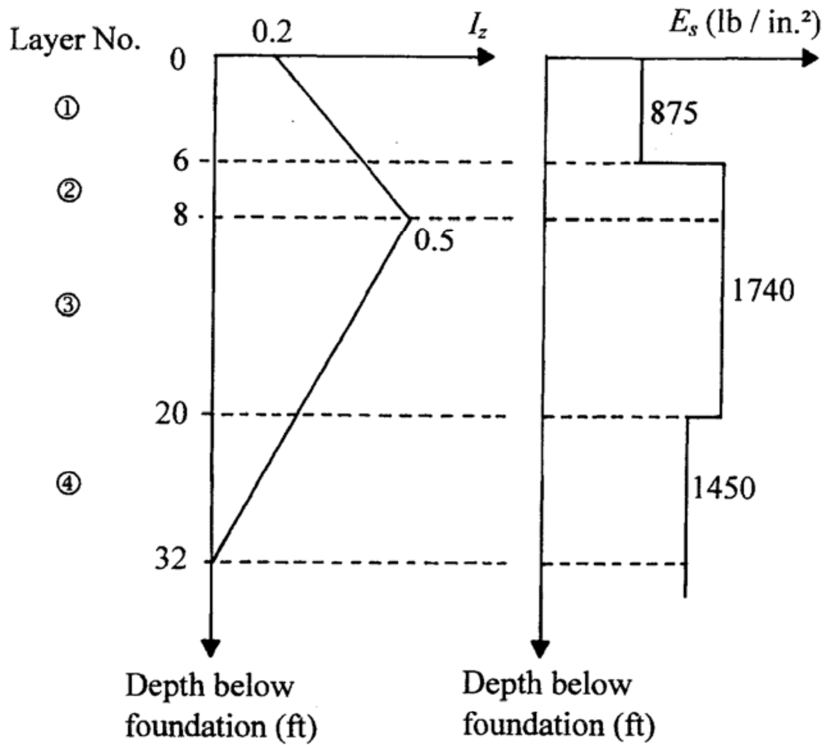
From Eq. (5.41):

$$I_E = 1 - \frac{1}{3.5\exp(1.22\mu_s - 0.4)\left(\dfrac{B_e}{D_f} + 1.6\right)} = 1 - \frac{1}{3.5\exp[(1.22)(0.3) - 0.4]\left(\dfrac{3.385}{1.5} + 1.6\right)}$$

$$= 0.923$$

$$S_e = (150)(3.385)(0.89)(0.815)(0.923)\left(\frac{1-0.3^2}{16{,}000}\right) = 0.0193 \text{ m} = \mathbf{19.3\ mm}$$

5.18 See the figure below for strain influence factor diagram.



Depth (ft)	Δz (in.)	E_s (lb / in.²)	I_z	$\frac{I_z}{E_s}(\Delta z)$
0–6	72	875	0.313	0.0258
6–8	24	1740	0.463	0.0064
8–20	144	1740	0.375	0.0301
20–32	144	1450	0.125	0.0124
				$\sum 0.0756$

4.375 E-5 ft³/lb

$q = \gamma D_f = (115)(5) = 575 \text{ lb / in.}^2$

$$C_1 = 1 - 0.5\left(\frac{q}{\bar{q} - q}\right) = 1 - 0.5\left(\frac{575}{4000 - 575}\right) = 0.916; \quad C_2 = 1 + 0.2\log\left(\frac{10}{0.1}\right) = 1.4$$

$$S_e = C_1 C_2 (\bar{q} - q)\Sigma \frac{I_z}{E_s}\Delta z = (0.916)(1.4)\left(\frac{4000 - 575}{144}\right)(0.0756) = \mathbf{2.31\ in.}$$

5.19

Depth (ft)	N_{60}
5	10
10	12
15	9
20	14
25	16

relevant region for $D_f = 3'$, $B = 5'$, $L = 5'$ (depths 5–15 ft)

Average $N_{60} \approx$ ~~12~~ 10

From Eq. (5.46b):

$$\text{Allowable } q_{\text{net}} = \frac{N_{60}}{4}\left(\frac{B+1}{B}\right)^2 F_d\, S_e$$

$B = 5$ ft; $S_e = 1$ in.

$F_d = 1 + 0.33(D_f/B) = 1 + (0.33)(3/5) = 1.198$

$$q_{\text{net}} = \frac{\not{12}\ 10}{4}\left(\frac{5+1}{5}\right)^2 (1.198)(1) \approx \not{5.18}\ 4.31 \text{ kip / ft}^2$$

5.23 $\sigma'_o = (4.5)(100) + (3)(122 - 62.4) + \frac{10}{2}(120 - 62.4)$

$= 450 + 178.8 + 288 = 916.8 \text{ lb / ft}^2$

$\sigma'_o + \Delta\sigma'_{av} = 916.8 + \not{1686} \to 812.8 = \not{2602.8} \to 1730 \text{ lb / ft}^2 = \sigma'_{vf}$

$$S_{e(p)} = \frac{C_s H_c}{1+e_o}\log\frac{\sigma'_{vf}}{\sigma'_o} + \not{\frac{C_c H_c}{1+e_o}\log\frac{\sigma'_o + \Delta\sigma'_{av}}{\sigma'_c}}$$

$$= \frac{(0.06)(10\times 12)}{1+0.7}\log\left(\frac{\not{2000}\ 1730}{916.8}\right) + \not{\frac{(0.25)(10\times 12)}{1+0.7}\log\left(\frac{2602.8}{2000}\right) = 3.45 \text{ in.}}$$

$S_{e(p)} = 1.17$ in

5.24 From Problems 5.23 and 5.7

$\sigma'_o = 916.8 \text{ lb/ft}^2$

$\sigma'_o + \Delta\sigma' = 916.8 + 706.3 = 1623.1 \text{ lb/ft}^2$

$$S_{e(p)} = \frac{C_s H_c}{1+e_o}\log\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o} = \frac{(0.06)(10)}{1.7}\log\frac{1623.1}{916.8} = 0.0875 \text{ ft} = \mathbf{1.05\ in.}$$

Solution to Homework Assignment #5 Supplemental Problem:

Problem Statement:

Need to design a foundation system for an elevated 30'x30'x30' water storage tank shown in the Figure below. For the weight of the structure (dead load) use a load factor of 1.4, and for the live loads on the structure from winds, use a load factor of 1.7.

Compute the minimum permissible size of a shallow square footing which will serve as the foundation for the tank such that:

1. A factor of safety against bearing failure of at least 3 is achieved;
2. Under the factored design loads, the settlement of the tank's foundation is no more than one inch;

Notes:

- The soil is granular with a friction angle of 35º, and a unit weight of 120 pcf;
- The depth of the foundation can be taken as 6 ft.;
- Assume that when the 30'x30'x30' tank is full, water constitutes 65% of the total weight of the structure.

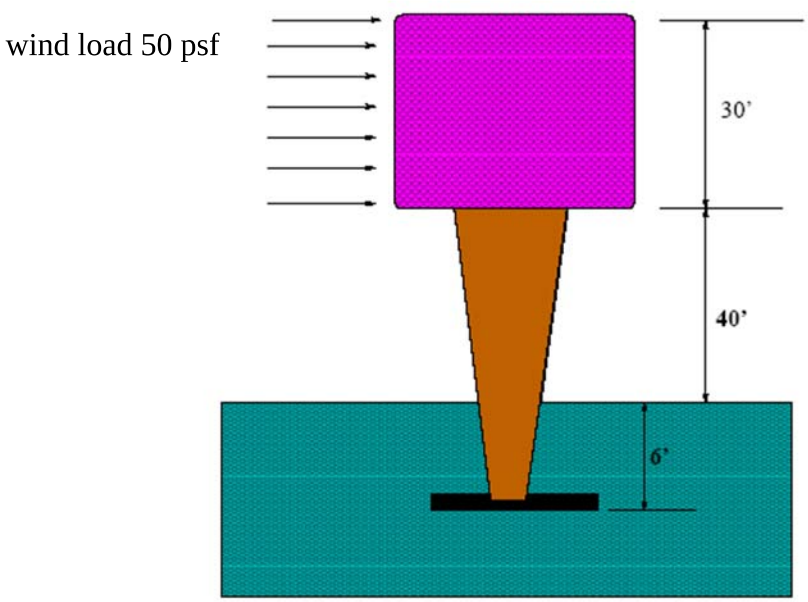


Figure 1: Schematic of an elevated water tank and its foundation system.

SOLUTION

A. Loads:

$$\text{Weight of water} = (30\text{ ft})^3 * 62.4 lbs \cdot ft^{-3} = 1.685 \cdot 10^6 lbs$$

$$\text{Weight of structure \& water} = \frac{\text{Weight of water}}{0.65} = 2.592 \cdot 10^6 lbs$$

$$\text{Factored Dead Load} = \text{Q} = 1.4 * 2.592 \cdot 10^6 lbs = 3.6 \cdot 10^6 lbs$$

$$\text{Factored wind load} = \text{H} = 1.7 * (30\text{ ft})^2 * 50 lbs \cdot ft^{-2} = 7.65 \cdot 10^4 lbs$$

$$\text{Factored wind load moment} = \text{H} * (6\text{ft} + 40\text{ft} + 15\text{ft}) = 4.67 \cdot 10^6 lb \cdot ft$$

$$\text{Eccentricity } e = \frac{\text{M}}{\text{Q}} = \frac{4.67 \cdot 10^6 lb \cdot ft}{3.6 \cdot 10^6 lbs} = 1.3 ft.$$

$$\text{Resultant magnitude of tower load} = (\text{Q}^2 + H^2)^{1/2} = 3.6 \cdot 10^6 lbs$$

$$\text{Inclination angle } \beta = \sin^{-1}\left(\frac{H}{Q}\right) = 1.27^\circ$$

B. Sizing of Foundation Based on Bearing Capacity

Should have $Q = 3.6 \cdot 10^6 lbs \le Q'_{all} = \dfrac{Q'_u}{FS} = \dfrac{q'_u * B'L'}{FS}$

Use the general bearing capacity equations to solve for q'_u

For $\phi = 35^\circ$, $\text{N}_\text{q} = 33.3$; $\text{N}_\gamma = 48.03$

$$B' = B - 2e = B - 2.6 ft;$$

$$L' = B$$

$$F_{qs} = 1 + \left(\frac{B'}{L'}\right)\tan\phi = 1 + \left(\frac{B - 2.6'}{B}\right) * 0.70 = 1.70 - \frac{1.82'}{B}$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B'}{L'}\right) = 1 - 0.4\left(\frac{B - 2.6'}{B}\right) = 0.6 + \frac{1.04'}{B}$$

Assuming that $\dfrac{D_f}{B} < 1$:

$$F_{qd} = 1 + 2\tan\phi(1 - \sin\phi)^2 \frac{D_f}{B} = 1 + 0.254\frac{D_f}{B} = 1 + \frac{1.524'}{B}$$

$$F_{\gamma d} = 1$$

$$q_u' = qN_q F_{qs} F_{qd} + \frac{\gamma B'}{2} N_\gamma F_{\gamma s} F_{\gamma d}$$

$$= (720\,psf) * 33.3 * \left(1.70 - \frac{1.82'}{B}\right)\left(1 + \frac{1.524'}{B}\right) + \frac{120\,pcf * (B - 2.6')}{2} * 48.03 * \left(0.6 + \frac{1.04'}{B}\right)$$

Since we need to have: $q_u' = \frac{Q * FS}{B'L'} = \frac{10.8 \cdot 10^6\,lbs}{(B - 2.6')B}$

$$\frac{10.8 \cdot 10^6\,lbs}{(B - 2.6')B} = (720\,psf) * 33.3 * \left(1.70 - \frac{1.82'}{B}\right)\left(1 + \frac{1.524'}{B}\right) + \frac{120\,pcf * (B - 2.6')}{2} * 48.03 * \left(0.6 + \frac{1.04'}{B}\right)$$

This can be algebraically simplified to yield the following quartic equation for B:

$$B^4 + (25.304\,ft)B^3 - (52.85\,ft^2)B^2 - (6300\,ft^3)B + 100.13\,ft^4 = 0$$

The meaningful root to this equation is: $B = 13.45\,ft \Rightarrow 13.5\,ft$.

C. Sizing of the Foundation for Settlements

- The sand has a unit weight of 120 pcf. If the unit weight of 120 pcf corresponds to dry soil with no moisture content, it would represent a very densely packed sand, since the estimated void ratio is: $e = \frac{G_s \gamma_w}{\gamma_d} - 1 = \frac{2.72 * 62.4\,pcf}{120\,pcf} - 1 = 0.41$.

- On the other hand, if the unit weight of 120 pcf has some moisture in it, the void ratio could be somewhat larger.

- To be conservative, we'll say the sandy soil is moderately compacted. From Table 5.8 of the textbook, the range of Young's modulus for a medium dense sand is 2500 – 4000 psi. To be conservative, let's choose a value of E=2500 psi and a Poisson's ratio of 0.35.

- Assume the sand layer beneath the water tower is infinite. We'll use the Schmertmann solution for elastic settlement which is: $S_e = C_1 C_2 q_{net} \sum \frac{I_z}{E_s} \Delta z$

- Here, the Young's modulus is assumed constant with depth, so we can re-write this as:

$$S_e = \frac{C_1 C_2 q_{net}}{E_s} \sum I_z \Delta z$$

- For a square footing, $\sum I_z \Delta z = .525B$
- $q_{net} = \frac{Q}{A} - \gamma D_f = \frac{3.6 \cdot 10^6 \, lbs}{(13.5\,ft)^2} - 120\,pcf * 6\,ft = 19{,}753\,psf - 720\,psf = 19{,}033\,psf$
- $C_1 = 1 - \frac{0.5\gamma D_f}{q_{net}} = 0.981$
- $C_2 = 1 + 0.2\log(10 * t_{years})$; Assuming t = 25yrs, $C_2 = 1.48$
- $E = 2500\,psi \Rightarrow 3.6 \cdot 10^5 \, psf$
- $S_e = \frac{(0.981)(1.48)(1.9 \cdot 10^4 \, psf)(.525 * 13.5\,ft)}{3.6 \cdot 10^5 \, psf} = 0.54\,ft = 6.5''$
- This settlement is far too large!! So the foundation size must be increased to get the net bearing stress way down. This is done by trial and error in the table shown below:

B(ft)	E_s(psf)	q(psf)	q_{net}(psf)	C_1	C_2	I*B(ft)	S_e(ft)	S_e(in)
13.5	3.60E+05	7.20E+02	1.90E+04	9.81E-01	1.48	7.0875	5.44E-01	6.53E+00
20	3.60E+05	7.20E+02	8.28E+03	9.57E-01	1.48	10.5	3.42E-01	4.10E+00
30	3.60E+05	7.20E+02	3.28E+03	8.90E-01	1.48	15.75	1.89E-01	2.27E+00
40	3.60E+05	7.20E+02	1.53E+03	7.65E-01	1.48	21	1.01E-01	1.21E+00
45	3.60E+05	7.20E+02	1.06E+03	6.60E-01	1.48	23.625	6.78E-02	8.13E-01

So, based on settlement considerations, the minimum acceptable foundation size is B=45ft. In this design problem, settlement is clearly the controlling factor.

Since the sizing of the foundation is based entirely on conservative estimates of the soil's Young's modulus E_s, a subsurface soil investigation that explicitly gives more information on the Young's modulus of the soil would be very helpful. A larger value of E_s based on actual site data could permit usage of a much smaller foundation size.