53:139 Foundation Engineering

Homework #6 Solutions

Spring Semester 2009

6.4 a. $B = 20$ m; $c_u = 30$ kN / m^2; $L = 20$ m; $\gamma = 18.5$ kN / m^3

$$\text{Eq. (6.20): } D_f = \frac{Q}{A\gamma} = \frac{48 \times 1000 \text{ kN}}{(20 \times 20)(18.5)} = \mathbf{6.49\ m}$$

b. Eq. (6.22):

$$\text{FS} = \frac{5.14c_u\left(1+\dfrac{0.195B}{L}\right)\left(1+\dfrac{0.4D_f}{B}\right)}{\dfrac{Q}{A} - \gamma D_f}$$

$$2 = \frac{(5.14)(30)\left(1+\dfrac{(0.195)(20)}{20}\right)\left(1+\dfrac{0.4D_f}{20}\right)}{\left(\dfrac{48\times 10^3}{20\times 20}\right) - 18.5D_f}$$

$$240 - 37D_f = 184.27 + 3.69D_f;\ D_f = \mathbf{1.37\ m}$$

6.6 $B = 10$ m; $L = 12$ m; $Q = 30{,}000$ kN

Eq. (5.70): $\Delta\sigma'_{av} = \frac{1}{6}(\Delta\sigma'_t + \Delta\sigma'_m + \Delta\sigma'_b)$

Eq. (5.12): $m_1 = L/B = 12/10 = 1.2$

z (m)	$n_1 = \frac{z}{\left(\frac{B}{2}\right)}$	m_1	I_c (Table 5.3)	$\Delta\sigma' = q_o I_c$ (kN / m²)
4	0.8	1.2	≈ 0.81	$0.81\left(\frac{30 \times 1000}{10 \times 12}\right) = 202.5$
6.6	1.32	1.2	≈ 0.64	$0.64\left(\frac{30 \times 1000}{10 \times 12}\right) = 160$
9.2	1.84	1.2	≈ 0.45	$0.45\left(\frac{30 \times 1000}{10 \times 12}\right) = 112.5$

$$\Delta\sigma'_{av} = \frac{1}{6}(202.5 + 4 \times 160 + 112.5) = 159.2 \text{ kN / m}^2$$

$$S_c = \frac{C_c H_c}{1 + e_o} \log \frac{\sigma'_o + \Delta\sigma'_{av}}{\sigma'_o}$$

$$\sigma'_o = (16)(4.2) + 2(18 - 9.81) + 2.6(17.5 - 9.81) = 103.58 \text{ kN / m}^2$$

$\sigma'_c = 105$ kN / m². So $\sigma'_o \approx \sigma'_c$ (normally consolidated).

$$S_c = \frac{(0.38)(5.2)}{1 + 0.88} \log \frac{103.58 + 159.2}{103.58} = \mathbf{0.425\ m}$$

6.9 Eq. (6.24): $q = \dfrac{Q}{A} \pm \dfrac{M_y x}{I_y} \pm \dfrac{M_x y}{I_x}$

$A = (16.5)(21.5) = 354.75 \text{ m}^2$

$I_x = \dfrac{1}{12} BL^3 = \dfrac{1}{12}(16.5)(21.5)^3 = 13{,}665 \text{ m}^4$

$I_y = \dfrac{1}{12} LB^3 = \dfrac{1}{12}(21.5)(16.5)^3 = 8{,}050 \text{ m}^4$

$Q = 350 + \cancel{(2)}(400) + \overset{2*}{450} + (2)(500) + (2)(1200) + (4)(1500) = 11{,}0\overset{5}{\cancel{0}}0 \text{ kN}$

$M_y = Qe_x; \quad e_x = x' - \dfrac{B}{2}$

$x' = \dfrac{Q_1 x_1' + Q_2 x_2' + Q_3 x_3' + \dots}{Q}$

$$= \frac{1}{11{,}0\overset{5}{\cancel{0}}0}\begin{bmatrix} (8.25)(500+1500+1500+500) \\ +(16.25)(350+1200+1200+450) \\ +(0.25)(\underset{450}{\cancel{400}}+1500+1500+400) \end{bmatrix} = \overset{7.78}{\cancel{7.814}} \text{ m}$$

$e_x = x' - \dfrac{B}{2} = \underset{7.78}{\cancel{7.814}} - 8.25 = \cancel{-0.435 \text{ m} \approx -0.44 \text{ m}}$ -0.47 m.

Hence, the resultant line of action is located to the left of the center of the mat. So

$M_y = 5193.5$

$M_y = (11{,}000)(0.44) = \cancel{4840 \text{ kN}}$ - m. Similarly

$M_x = Qe_y; \quad e_y = y' - \dfrac{L}{2}$

$y' = \dfrac{Q_1 y_1' + Q_2 y_2' + Q_3 y_3' + \dots}{Q}$

$$= \frac{1}{\underset{11050}{\cancel{11{,}000}}}\begin{bmatrix} (0.25)(400+500+350)+(7.25)(1500+1500+1200) \\ +(14.25)(1500+1500+1200)+(21.25)(\underset{450}{\cancel{400}}+500+450) \end{bmatrix} = \cancel{10.85 \text{ m}} \ 10.89 \text{ m}$$

$e_y = y' - \dfrac{L}{2} = 10.85 - 10.75 = 0.1 \text{ m} \rightarrow 0.14 \text{ m}.$

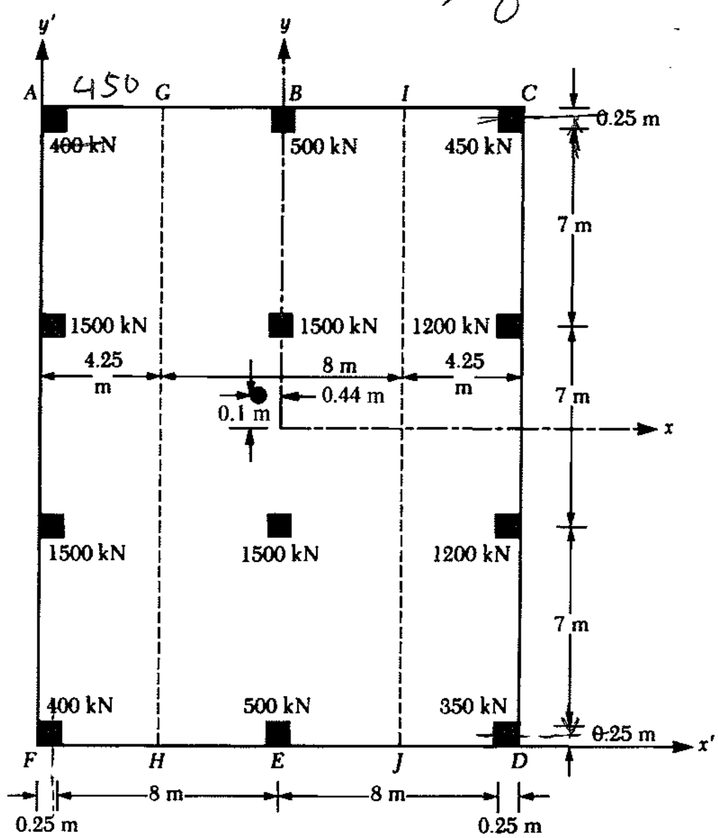
The location of the line of action of the resultant column loads is shown on the following page.

$M_x = 1547$

$M_x = (11{,}000)(0.1) = 1100$ kN - m. So

$$q = \frac{11{,}000}{354.75} \pm \frac{4840x}{8050} \pm \frac{1100y}{13{,}665} = 31.0 \pm 0.6x \pm 0.08y \text{ (kN / m}^2\text{)}$$

$\Rightarrow q = 31.15 \pm 0.645x \pm 0.113y$



At *A*: $q = 31.0 + (0.6)(8.25) + (0.08)(10.75) = \mathbf{36.81\ kN / m^2}$ 37.52

At *B*: $q = 31.0 + (0.6)(0) + (0.08)(10.75) = \mathbf{31.86\ kN / m^2}$ 32.36

At *C*: $q = 31.0 - (0.6)(8.25) + (0.08)(10.75) = \mathbf{26.91\ kN / m^2}$ 27.19

At *D*: $q = 31.0 - (0.6)(8.25) - (0.08)(10.75) = \mathbf{25.19\ kN / m^2}$ 24.77

At *E*: $q = 31.0 + (0.6)(0) - (0.08)(10.75) = \mathbf{30.14\ kN / m^2}$ 29.94

At *F*: $q = 31.0 + (0.6)(8.25) - (0.08)(10.75) = \mathbf{35.09\ kN / m^2}$ 35.11

6.10 Determination of Shear and Moment Diagrams for Strips:

Strip *AGHF*:

Average soil pressure $= q_{av} = q_{(at\,A)} + q_{(at\,F)} = \dfrac{36.81 + 35.09}{2} = 35.95\ \text{kN/m}^2$

Total soil reaction $= q_{av} B_1 L = (35.95)(4.25)(21.50) = 3285\ \text{kN}$

Average load $= \dfrac{\text{load soil reaction} + \text{column loads}}{2} = \dfrac{3285 + 3800}{2} = 3542.5\ \text{kN}$

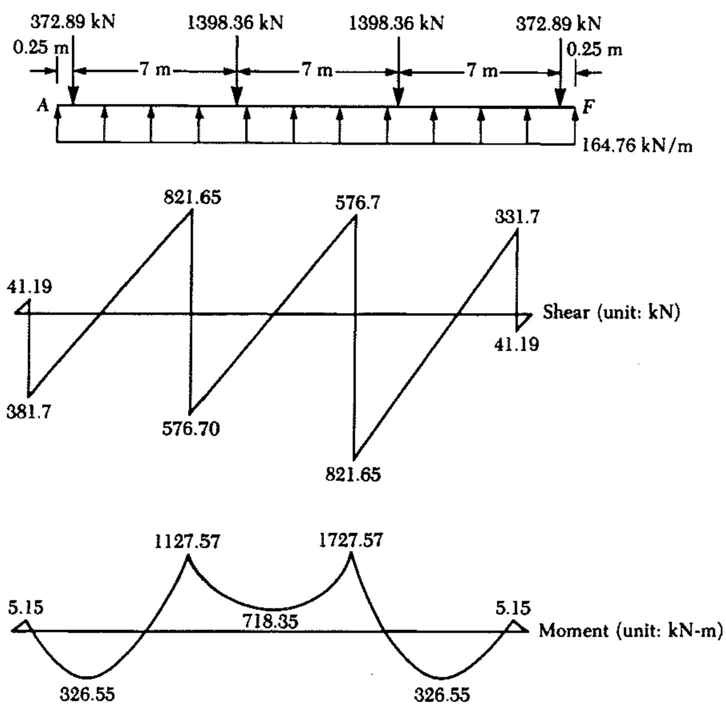
So, modified average soil pressure,

$$q_{av(modified)} = q_{av}\left(\frac{3542.5}{3285}\right) = (35.95)\left(\frac{3542.5}{3285}\right) = 38.768\ \text{kN/m}^2$$

The column loads can be modified in a similar manner by multiplying factor

$$F = \frac{3542.5}{3800} = 0.9322$$

Figure (a) shows the loading on the strip and corresponding shear and moment diagrams. Note that the column loads shown in this figure have been multiplied by $F = 0.9322$. Also the load per unit length of the beam is equal to $B_1\, q_{av(modified)} = (4.25)(38.768) = 164.76$ kN / m.



Strip *GIJH*: In a similar manner

$$q_{av} = \frac{q_{(\text{at } B)} + q_{(\text{at } E)}}{2} = \frac{31.86 + 30.14}{2} = 31.0 \text{ kN / m}^2$$

Total soil reaction = (31)(8)(21.5) = 5332 kN

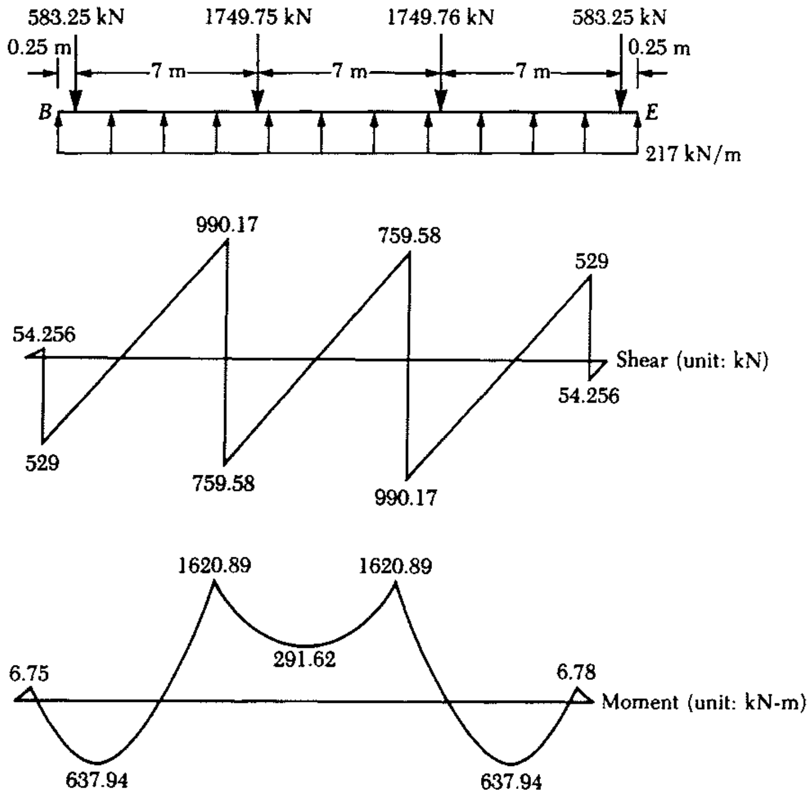
Total column load = 4000 kN

$$\text{Average load} = \frac{5332 + 4000}{2} = 4666 \text{ kN}$$

$$q_{av(\text{modified})} = (31.0)\left(\frac{4666}{5332}\right) = 27.12 \text{ kN / m}^2$$

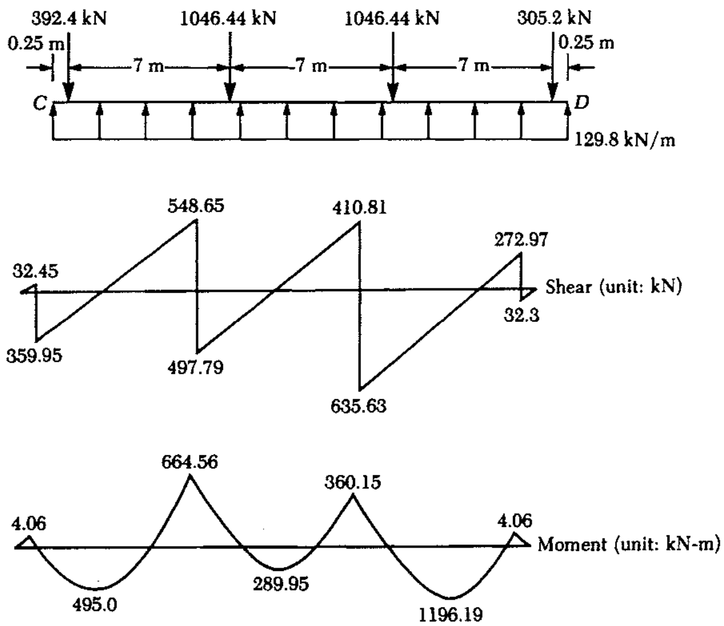
$$F = \frac{4666}{4000} = 1.1665$$

The load, shear, and moment diagrams are shown in Figure (b).



(b) Strip GIJH

Strip *ICDJ*: Figure (c) shows the load, shear, and moment diagrams for this strip.



(c) Strip ICDJ

6.11 Eq. (6.45): $k = k_1\left(\frac{B+1}{2B}\right)^2 = 55\left(\frac{25+1}{50}\right)^2 = \mathbf{14.9\ lb / in.^3}$

6.12 $B = 30$ ft; $L = 70$ ft. Eq. (6.48): $k = \dfrac{k_{(B\times B)}\left(1+0.5\dfrac{B}{L}\right)}{1.5}$

$K_{B\times B} = 14.68$

~~From Problem 6.11, $k_{(B\times B)} = 14.9$ lb / in.³~~

$$k = \frac{\overset{14.68}{(\cancel{14.9})}\left[1+0.5\left(\frac{30}{70}\right)\right]}{1.5} = \overset{11.9}{\cancel{12.1}}\ \mathbf{lb / in.^3}$$

6.13 From Eq. (6.48):

$$\cancel{\frac{k_{(1\times 0.7)}}{k_{(5\times 3.5)}} = \frac{k_{(B\times B)}\left[1+0.5\left(\frac{0.7}{1}\right)\right]}{k_{(B\times B)}\left[1+0.5\left(\frac{3.5}{5}\right)\right]} = \frac{1.35}{1.35} = 1}$$

$$\cancel{k_{(5\times 3.5)} = \frac{k_{(1\times 0.7)}}{1} = \frac{18}{1} = \mathbf{18\ kN / m^3}}$$

$$\frac{K_{(1\times 0.7)}}{K_{(5\times 3.5)}} = \frac{K_{(0.7\times 0.7)}}{K_{(3.5\times 3.5)}} \cdot 1 = \frac{K_{0.3}\cdot\left(\frac{0.7+0.3}{2\times 0.7}\right)^2}{K_{0.3}\cdot\left(\frac{3.5+0.3}{3.5*2}\right)^2} = 1.73$$

$$\Rightarrow K_{(5\times 3.5)} = 10.4\ kN/m^3$$

Supplemental Problem:

Consider the mat foundation resting on a system of elastic springs. For the single column load shown, use Approximate Flexible Method presented in the textbook to compute the moments M_r and M_t, shear V and deflection δ as a function of distance from the column load, at r = 0; 0.5L'; L'; 2L'; 4L'; and 6L'.

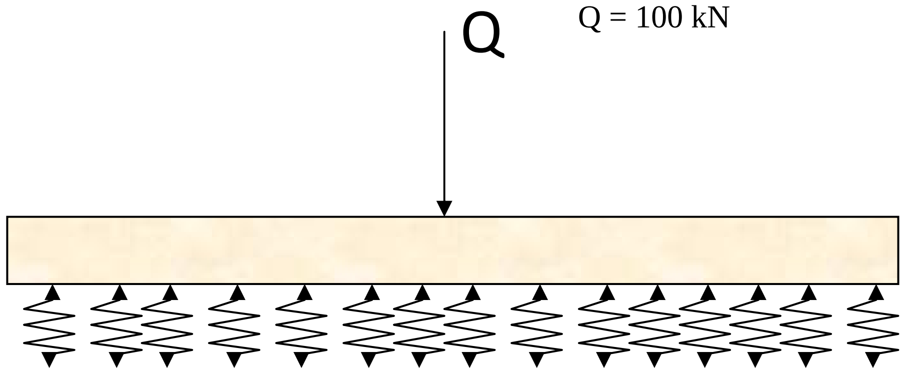
E_c = 21 Gpa;

ν_c = 0.28;

h = 1m;

k_s = 25MNm^{-3}

Q = 100 kN



Solution:. The solution is given for k_s=25MN/m^3.

a) Solution for k_s=25MN/m^3:

$$M_r = \frac{-Q}{4}\left[Z_4 - \frac{(1-\nu_c)Z_3'}{(\frac{r}{L'})}\right] \qquad D = \frac{E_c h^3}{12(1-\nu_c^2)}$$

$$M_\theta = \frac{-Q}{4}\left[\nu_c Z_4 - \frac{(1-\nu_c)Z_3'}{(\frac{r}{L'})}\right] \qquad L = \left(\frac{D}{k_s}\right)^{1/4}$$

$$V = \frac{-Q}{4L'}Z_4'$$

$$\delta = \frac{Q(L')^2 Z_3}{4D}$$

$$D = \frac{E_c h^3}{12(1-\nu_c{}^2)} = \frac{(21\cdot10^9\,Pa)(1m^3)}{12(1-0.28^2)} = 1.90\cdot10^9\,N\cdot m$$

$$L' = \left(\frac{D}{k_s}\right)^{0.25} = \left(\frac{1.90\cdot10^9\,N\cdot m}{25\cdot10^6\,N\cdot m^{-3}}\right)^{0.25} = 2.95m$$

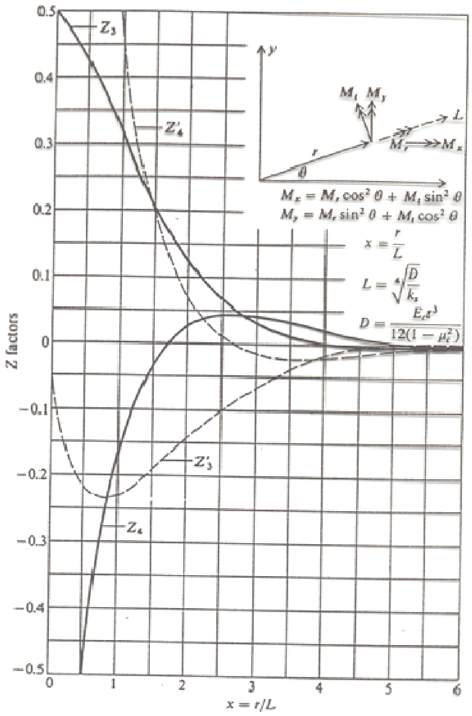


FIGURE 10-6 Z_i factors for computing deflections, moments and shears in a flexible plate. [*After Hetenyi (1946).*]

Taken From: Foundation Analysis and Design, 4th Ed.
J. E. Bowles
McGraw-Hill 1988.

Solution for k_s=25MN/m3									
L'=2.95m; Q=10^5N; ν_c=0.28; D=$1.90\cdot10^9$N·m									
r(m)	r/L'	Z_3	Z_4	Z_3'	Z_4'	M_r(kN)	M_θ(kN)	V(kN/m)	δ(m)
0	0	0.5	undef.	-0.05	undef.	undef.	undef.	undef.	5.73E-05
1.475	0.50	0.42	-0.5	-0.22	1	-9.89	-4.17	-8.47	4.81E-05
2.95	1.00	0.315	-0.17	-0.23	0.45	-2.89	-2.87	-3.81	3.61E-05
5.9	2.00	0.12	-0.03	-0.145	0.07	-0.31	-1.08	-0.59	1.37E-05
11.8	4.00	0.0	0.02	-0.015	-0.02	0.52	-0.22	0.17	0.0
17.7	6.00	0.0	0.0	0.0	0.0	0.00	0.0	0.0	0.0