53:139 Foundation Engineering

Homework #8 Solutions

Spring Semester 2009

53:139 Foundation Engineering
Department of Civil & Environmental Engineering
The University of Iowa
Assignment #8 Solution

Book Problems: From Chapter 9, solve problems: 1, 5, 8, 9, 16, and 17.

Supplemental Problem #1:

For a vertical retaining wall of height H and a sloping granular backfill of unit weight γ and friction angle ϕ, making angle α with respect to the horizontal, the resultant Rankine active and passive forces acting on the wall are given by the following relations:

$$P_a = \frac{\gamma H^2}{2}\cos\alpha\left[\frac{\cos\alpha - \left(\cos^2\alpha - \cos^2\phi\right)^{1/2}}{\cos\alpha + \left(\cos^2\alpha - \cos^2\phi\right)^{1/2}}\right]$$

$$P_p = \frac{\gamma H^2}{2}\cos\alpha\left[\frac{\cos\alpha + \left(\cos^2\alpha - \cos^2\phi\right)^{1/2}}{\cos\alpha - \left(\cos^2\alpha - \cos^2\phi\right)^{1/2}}\right]$$

The resultant forces act parallel to the sloping granular backfill that makes angle α with respect to the horizontal. **Derive both of these results using fundamental principles**.

Supplemental Problem #2:

In Section 9.7 of the textbook, the special case of a cantilever wall penetrating a saturated clay soil is considered. Using equilibrium considerations derive Eqs. (9.52) – (9.55). Show all of your work with labeled sketch.

9.1 a. Refer to Figure 9.7 in the text. $L_1 = 4$ m; $L_2 = 8$ m

$\gamma = 16.1 \text{ kN/m}^3$; $\gamma_{sat} = 18.2 \text{ kN/m}^3$; $\phi = 32°$

$\gamma' = 18.2 - 9.81 = 8.39 \text{ kN/m}^3$

$$K_a = \tan^2\left(45 - \frac{32}{2}\right) = 0.307; \quad K_p = \tan^2\left(45 + \frac{32}{2}\right) = 3.255$$

$$\sigma'_1 = \gamma L_1 K_a = (16.1)(4)(0.307) = 19.77 \text{ kN/m}^2$$

$$\sigma'_2 = (\gamma L_1 + \gamma' L_2)K_a = [(16.1)(4) + (8.39)(8)](0.307) = 40.38 \text{ kN/m}^2$$

$$L_3 = \frac{40.38}{(8.39)(3.255 - 0.307)} = 1.63 \text{ m}$$

$$P = \frac{1}{2}(4)(19.77) + (8)(19.77) + \frac{1}{2}(8)(40.38 - 19.77)$$
$$+ \frac{1}{2}(1.63)(40.38) = 39.54 + 158.16 + 82.44 + 32.91 = 313.05 \text{ kN/m}$$

$$P(\bar{z}) = (39.54)\left(9.63 + \frac{4}{3}\right) + (158.16)(5.63) + (82.44)\left(1.63 + \frac{8}{3}\right)$$
$$+ \left(\frac{(2)(1.63)}{3}\right)(32.91)$$

$$\bar{z} = \frac{1713.91}{313.05} = 5.47 \text{ m}$$

$$\sigma'_5 = (\gamma L_1 + \gamma' L_2)K_p + \gamma' L_3 (K_p - K_a)$$
$$= [(16.1)(4) + (8.39)(8)](3.255) + (8.39)(1.63)(2.948) = 468.4 \text{ kN/m}^2$$

$$A_1 = \frac{468.4}{(8.39)(2.948)} = 18.94$$

$$A_2 = \frac{(8)(313.05)}{(8.39)(2.948)} = 101.25$$

$$A_3 = \frac{(6)(313.05)[(2)(5.47)(8.39)(2.948) + 468.4]}{(8.39)^2(2.948)^2} = 2268.94$$

$$A_4 = \frac{(313.05)[(6)(5.47)(468.4) + (4)(313.05)]}{(8.39)^2(2.948)^2} = 8507.44$$

$$L_4^4 + 18.94L_4^3 - 101.25L_4^2 - 2268.94L_4 - 8507.44 = 0; \; L_4 = 11.68 \text{ m}$$

$D = L_3 + L_4 = 1.63 + 11.68 =$ **13.31 m**

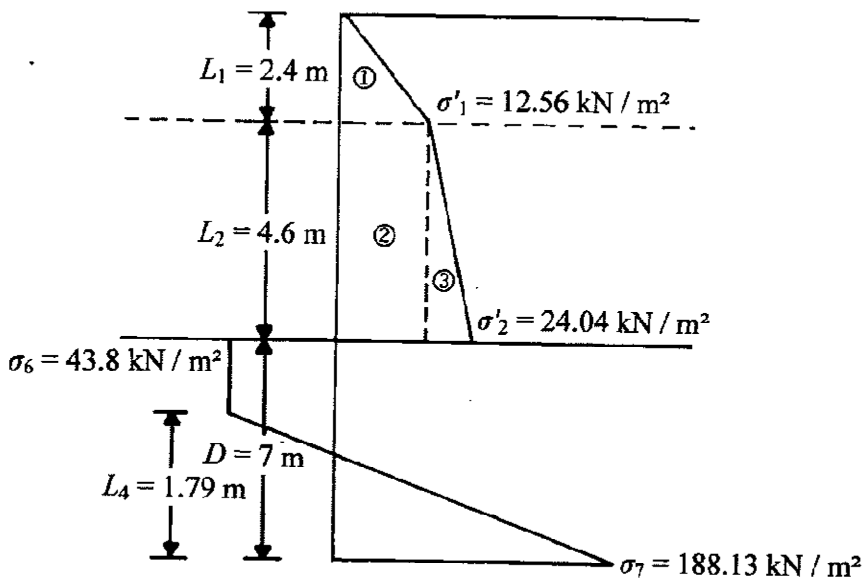
b. Total length = 4 + 8 + (1.3)(13.31) = **29.3 m**

c. $z' = \sqrt{\dfrac{2P}{\gamma'(K_p - K_a)}} = \sqrt{\dfrac{(2)(313.05)}{(8.39)(2.948)}} = 5 \text{ m}$

$$M_{max} = P(\bar{z} + z') - \frac{1}{6}\gamma' z'^3 (K_p - K_a)$$

$$= (313.05)(5.47 + 5) - \frac{1}{6}(8.39)(5)^3(2.948) = \mathbf{2762\ kN\text{-}m/m}$$

9.5 a. Refer to the following figure:



$$K_a = \tan^2\left(45 - \frac{\phi'}{2}\right) = \tan^2(45 - 15) = \frac{1}{3}$$

$$K_a = \tan^2\left(45 + \frac{\phi'}{2}\right) = \tan^2(45 + 15) = 3$$

$$\sigma'_1 = \gamma L_1 K_a = (15.7)(2.4)\tfrac{1}{3} = 12.56 \text{ kN/m}^2$$

$$\sigma'_2 = (\gamma L_1 + \gamma' L_2) = [(15.7)(2.4) + (17.3 - 9.81)(4.6)]\tfrac{1}{3} = 24.04 \text{ kN/m}^2$$

P_1 = Areas of 1 + 2 + 3 = 15.07 + 57.78 + 26.4 = 99.25 kN / m

$$\bar{z}_1 = \frac{(15.07)(5.4) + (57.78)(2.3) + (26.4)\left(\dfrac{4.6}{3}\right)}{99.25} = 2.567 \text{ m}$$

Eq. (9.44): $D^2[4c-(\gamma L_1+\gamma' L_2)]-2DP_1-\dfrac{P_1(P_1+12c\bar{z}_1)}{(\gamma L_1+\gamma' L_2)+2c}=0$

$$D^2[(4)(29)-72.13]-2D(99.25)-\frac{99.25[99.25+(12)(29)(2.567)]}{72.13+(2)(29)}=0$$

$43.87D^2 - 198.5D - 757 = 0$; $D =$ **7 m**

b. Length of sheet pile = 2.4 + 4.6 + 1.4(7) = **16.8 m**

c. Eq. (9.38): $\sigma_6 = 4c - (\gamma L_1 + \gamma' L_2)$

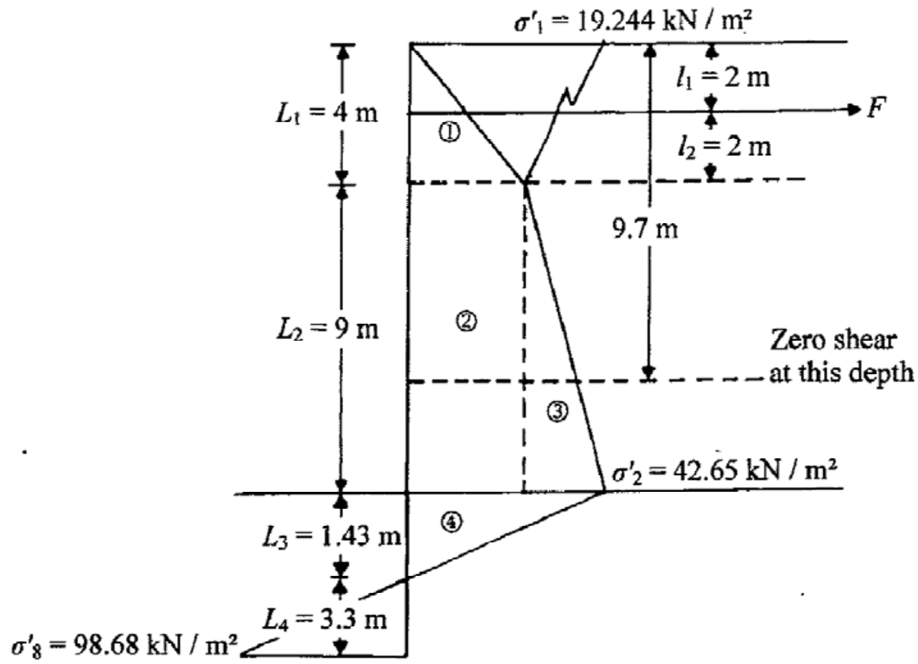
$$= (4)(29)-[(15.7)(2.4)-(17.3-9.81)(4.6)] = 43.87\ \text{kN/m}^2$$

Eq. (9.45): $z' = \dfrac{P_1}{\sigma_6} = \dfrac{99.25}{43.87} = 2.26$ m

$$M_{max} = P_1(z'+\bar{z}_1)-\frac{\sigma_6 z'^2}{2} = (99.25)(2.26+2.567)-\frac{(43.87)(2.26)^2}{2}$$

$= 479.08 - 112.04 =$ **367.04 kN - m / m of the wall**

9.8 a. Refer to the figure. $\phi' = 34°$.



$$K_a \tan^2\left(45 - \frac{\phi'}{2}\right) = 0.283; \quad K_p \tan^2\left(45 + \frac{\phi'}{2}\right) = 3.537$$

$$\sigma_1' = \gamma L_1 K_a = (17)(4)(0.283) = 19.244 \text{ kN / m}^2$$

$$\sigma_2' = (\gamma L_1 + \gamma' L_2) K_a = [(17)(4) + (19 - 9.81)(9)](0.283) = 42.65 \text{ kN / m}^2$$

$$L_3 = \frac{42.65}{\gamma'(K_p - K_a)} = \frac{42.65}{(9.19)(3.254)} \approx 1.43 \text{ m}$$

P = Area of 1 + 2 + 3 + 4 = 38.49 + 173.2 + 105.33 + 30.49 = 347.51 kN / m

$$\bar{z} = \frac{1}{347.51}\left[(38.49)(11.76) + (173.2)(5.93) + (105.33)(4.43) + (30.49)(0.95)\right] = 5.68 \text{ m}$$

Eq. (9.58):

$$L_4^3 + 1.5L_4^2(l_2 + L_2 + L_3) - \frac{3P[(L_1 + L_2 + L_3) - (\bar{z} + l_1)]}{\gamma'(K_p - K_a)} = 0$$

$$L_4^3 + 1.5L_4^2(2 + 9 + 1.43) - \frac{(3)(347.51)[(4 + 9 + 1.43) - (5.68 + 2)]}{(9.19)(3.254)} = 0$$

$L_4^3 + 18.645\, L_4^2 - 235.32 = 0;\; L_4 \approx 3.3$ m

$D = 3.3 + L_3 = 3.3 + 1.43 = \mathbf{4.73}$

b. Eq. (9.56): $\sigma_8' = \gamma' (K_p - K_a) L_4 = (9.19)(3.254)(3.3) \approx 98.68$ kN / m^2

The pressure distribution diagram is shown in the figure.

c. Eq. (9.57):

$$F = P - \frac{1}{2}[\gamma' (K_p - K_a) L_4^2] = 347.51 - \frac{1}{2}[(9.19)(3.254)(3.3)^2] = \mathbf{184.68\ kN / m}$$

9.9 a. $D_{actual} = (1.3)(D_{theory}) = (1.3)(4.73) \approx 6.15$ m

For zero shear, use Eq. (9.60): $\frac{1}{2}\sigma_1' L_1 - F + \sigma_1'(z - L_1) + \frac{1}{2} K_a\, \gamma'(z - L_1)^2 = 0$

Let $z - L_1 = x$

$$\frac{1}{2}\sigma_1' L_1 - F + \sigma_1' x + \frac{1}{2} K_a\, \gamma' x^2 = 0$$

$$\frac{1}{2}(19.244)(4) - 184.68 + 19.244x + \frac{1}{2}(0.283)(9.19)x^2 = 0$$

$x^2 + 14.8x - 112.46 = 0;\; x \approx 5.7$ m

$z = x + L_1 = 5.7 + 4 = 9.7$ m

Taking the moment about the point of zero shear

$$M_{max} = -\frac{1}{2}\sigma_1' L_1\left(x + \frac{4}{3}\right) + F(x + 2) - \sigma_1'\left(\frac{x^2}{2}\right) - \frac{1}{2} K_a \gamma' x^2 \left(\frac{x}{3}\right)$$

With $\sigma_1' = 19.244$ kN / m, $L_1 = 4$, $x = 5.7$, $F = 184.68$ kN / m

$M_{max} \approx$ **759 kN - m / m**

b. $H' = L_1 + L_2 + D_{actual} = 4 + 9 + 6.15 = 19.15$ m ≈ 20 m

Section	I (m^4/m)	H' (m)	$\rho = \frac{10.91 \times 10^{-7} H'^4}{EI}$	$\log \rho$	S (m^3/m)	$M_d = S\sigma_{all}$ (kN / m - m)	$\frac{M_d}{M_{max}}$
PZ-35	493.4×10^{-6}	20	16.85×10^{-4}	-2.773	260.5×10^{-5}	547.05	0.721
PZ-27	251.5×10^{-6}	20	33.05×10^{-4}	-2.481	162.3×10^{-5}	340.83	0.449

If $\log\rho$ and M_d/M_{max} are plotted in Figure 9.22 with the curve for loose sand as the separating line, it can be seen that **PZ-35 will be sufficient**; however, PZ-27 is not suitable.

9.16 From Figure 9.33(a), for $\phi' = 30°$, the value of $K_a \approx 0.31$.

$$W = Ht\gamma_{concrete} = (5)\left(\frac{3}{12}\right)(150) = 187.5 \text{ lb / ft}$$

Eq. (9.79):

$$K_p \sin\delta' = \frac{W + \frac{1}{2}\gamma H^2 K_a \sin\phi'}{\frac{1}{2}\gamma H^2} = \frac{187.5 + \frac{1}{2}(110)(5)^2(0.31)(0.5)}{\frac{1}{2}(110)(5)^2} = 0.291$$

From Figure 9.33(b), $K_p \cos\delta' \approx 3.4$

Eq. (9.78):

$$P_u' = \frac{1}{2}\gamma H^2 (K_p \cos\delta' - K_a \cos\phi') = \frac{1}{2}(110)(5)^2[3.4 - (0.31)(0.866)] = 4305.9 \text{ lb / ft}$$

Assume loose sand.

Eq. (9.85): $$P_{us}' = \left(\frac{C_{ov} + 1}{C_{ov} + \frac{H}{h}}\right) P_u' = \left(\frac{14 + 1}{14 + \frac{5}{3}}\right)(4328.5) = 4144.3 \text{ lb / ft}$$

$$\frac{S'-B}{H+h} = \frac{7-4}{5+3} = 0.375.$$ Referring to Figure 9.35(b),

$$\frac{B_e - B}{H+h} \approx 0.19; B_e = (0.19)(8) + 4 = 5.52$$

$$P_u = P'_{us} B_e = (4122.7)(5.52) \approx 22{,}757 \text{ lb} = \mathbf{22.76 \text{ kip}}$$

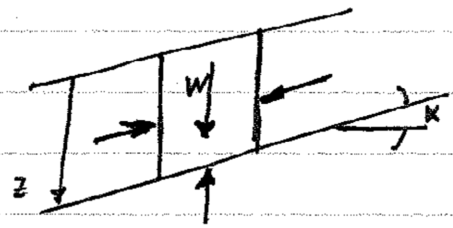
9.17 Eq. (9.82):

$$P_u = \frac{5.4}{\tan\phi'}\left(\frac{H^2}{Bh}\right)^{0.28} \gamma BhH = \frac{5.4}{\tan 32}\left(\frac{0.9^2}{0.3B}\right)^{0.28}(17)(B)(0.3)(0.9) = 52.39B^{0.72} \text{ kN}$$

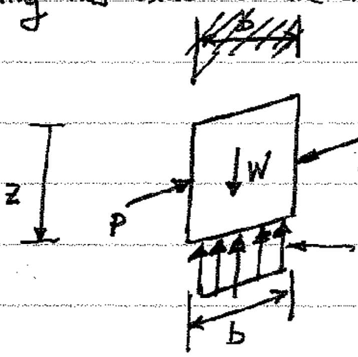
B (m)	$P_u = 52.39B^{0.72}$ (kN)
0.3	**22**
0.6	**36.3**
0.9	**48.6**

Bonus Problem Solution:

Consider for a moment an infinite granular slope



Isolating this slice from the slope and drawing its FBD we have:

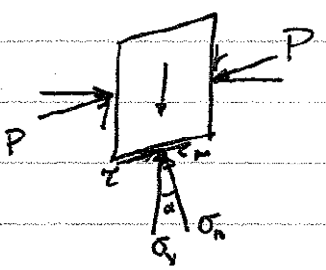


$W = \gamma z b \cos(\alpha)$

$\sigma_v = \frac{W}{b} = \gamma z \cos(\alpha)$

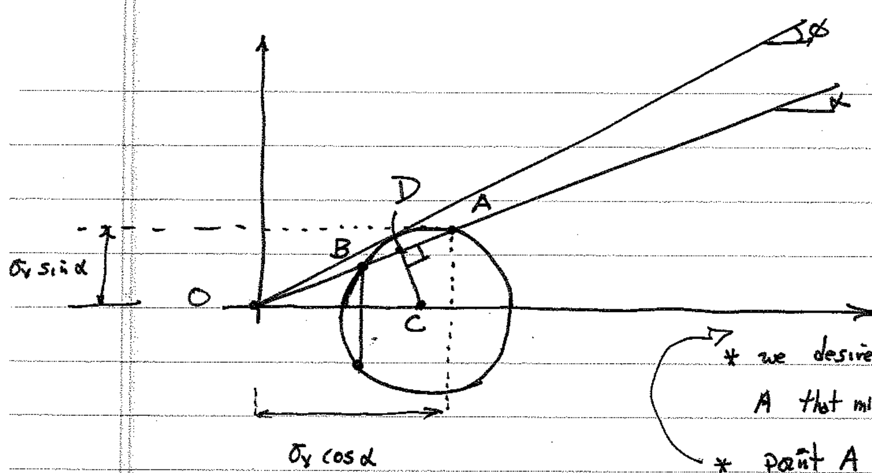
The force P is the one that would be exerted on a retaining wall, were one present.

To solve for P in the presence of soil failure, we resort to a Mohr's circle representation:



$\tau = \sigma_v \sin\alpha$

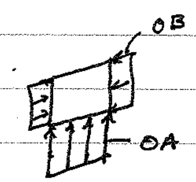
$\sigma_n = \sigma_v \cos\alpha$

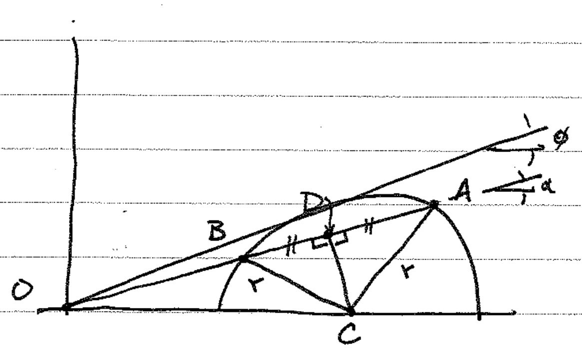


* we desire the M.C. passing through A that minimizes ?
* point A represents the stress state on planes ∥ to the slope at depth z
* pt. B is ∴ the so-called pole or origin of planes
* the stress at point B represents the stress state on vertical planes.

* magnitude of the (average) stress on vertical plane = $\overline{OB}$
* magnitude of stress resultant on slope plane = $\overline{OA}$

$$\text{ratio} = \frac{OB}{OA} = \frac{OD - BD}{OD + DA} =$$





$$BD = DA = \left(r^2 - \overline{DC}^2\right)^{1/2}$$

$$r = OC \sin\phi$$

$$DC = OC \sin\alpha$$

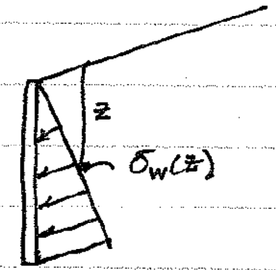
$$OD = OC \cos\alpha$$

$$\therefore \quad \frac{OB}{OA} = \frac{OC\cos\alpha - OC\left(\sin^2\phi - \sin^2\alpha\right)^{1/2}}{OC\cos\alpha + OC\left(\sin^2\phi - \sin^2\alpha\right)^{1/2}} = \frac{\cos\alpha - \left(\sin^2\phi - \sin^2\alpha\right)^{1/2}}{\cos\alpha + \left(\sin^2\phi - \sin^2\alpha\right)^{1/2}}$$

$$\frac{OB}{OA} = \frac{\cos\alpha - (\cos^2\alpha - \cos^2\phi)^{1/2}}{\cos\alpha + (\cos^2\alpha - \cos^2\phi)^{1/2}}$$

or: $\sigma_w = \sigma_v(z) \frac{OB}{OA}$

↑ wall stress



$$P_a = \int_0^H \sigma_w(z)\,dz = \int_0^H \sigma_v(z) \frac{OB}{OA}\,dz$$

$$= \frac{OB}{OA} \int_0^H \gamma z \cos\alpha\,dz = \frac{\gamma H^2}{2} \cos\alpha \frac{OB}{OA}$$

$$\therefore \boxed{P_a = \frac{\gamma H^2}{2} \cos\alpha \left[\frac{\cos\alpha - (\cos^2\alpha - \cos^2\phi)^{1/2}}{\cos\alpha + (\cos^2\alpha - \cos^2\phi)^{1/2}} \right]}$$

Using the same procedure and an alternative M.C. tangent to the failure envelope, we could also derive Rankine's passive stress on a vertical retaining wall with a granular backfill.

$$P_p = \frac{\gamma H^2}{2} \cos\alpha \left[\frac{\cos\alpha + (\cos^2\alpha - \cos^2\phi)^{1/2}}{\cos\alpha - (\cos^2\alpha - \cos^2\phi)^{1/2}} \right]$$