

53:139 Foundation Engineering

Homework #9 Solutions

Spring Semester 2009

11.1 a. Eq. (11.15): $Q_p = A_p q' N_q^*$

$$A_p = (0.46)^2 = 0.2116 \text{ m}^2$$

$$q' = \gamma L = (18.6)(20) = 372 \text{ kN / m}^2$$

$$N_q^* = 55 \text{ (Figure 11.12)}$$

$$Q_p = (0.2116)(372)(55) = \mathbf{4329 \text{ kN}}$$

Check: Eq. (11.17):

$$Q_p = A_p q_1 = 0.5 p_a A_p N_q^* \tan \phi' = (50)(0.2116)(55)(\tan 30) = 335.96 \text{ kN} \approx \mathbf{336 \text{ kN}}$$

Use $Q_p = \mathbf{336 \text{ kN}}$

b. Eq. (11.20) (for $c' = 0$): $Q_p = A_p \bar{\sigma}'_o N_\sigma^*$

$$I_{rr} = 75; \phi' = 30^\circ; N_\sigma^* = 45 \text{ (Table 11-5)}$$

$$\bar{\sigma}'_o = \left(\frac{1 + 2K_o}{3} \right) q'$$

$$q' = 372 \text{ kN / m}^2; K_o = 1 - \sin \phi' = 0.5$$

$$\bar{\sigma}'_o = \left[\frac{1 + (2)(0.5)}{3} \right] (372) = 248 \text{ kN / m}^2$$

$$Q_p = (0.2116)(248)(45) = 2361.5 \text{ kN} \approx \mathbf{2362 \text{ kN}}$$

c. Eq. (11.31) (for $c = 0$): $Q_p = A_p q' N_q^*$

$$N_q^* = 18.4 \text{ (Table 11.6)}$$

$$Q_p = (0.2116)(372)(18.4) = 1448.4 \text{ kN} \approx \mathbf{1448 \text{ kN}}$$

11.2 Eq. (11.37) $L' = 15D = (15)(0.46) = 6.9 \text{ m}$

At $z = 0$, $f = 0$.

At $z = 6.9 \text{ m}$, $f = K\sigma'_o \delta' = (1.5)(18.6 \times 6.9)[\tan(0.6 \times 30)] = 62.55 \text{ kN / m}^2$

$$Q_s(z=0 \text{ to } 6.9 \text{ m}) = (4 \times 0.46)(6.9)\left(\frac{62.55}{2}\right) = 397.1 \text{ kN}$$

$$Q_s(z=6.9 \text{ to } 20 \text{ m}) = (4 \times 0.46)(13.1)(62.55) = 1507.7 \text{ kN}$$

$$Q_s = 397.1 + 1507.7 \approx \mathbf{1905 \text{ kN}}$$

11.3 $\frac{L}{D} = \frac{20}{0.46} = 43.48$; $\phi' = 30^\circ$. From Figure 11.15, $N_q^* = 23$.

Eq. (11.33): $Q_p = A_p q' N_q^* = (0.2116)(372)(23) = 1810.5 \text{ kN}$

With $\frac{L}{D} = 43.48$, from Figure 11.19, $K \approx 0.2$ (extrapolation)

Eq. (11.41): $Q_s = K\bar{\sigma}'_o \tan(0.8\phi') pL$

$$= (0.2)\left(\frac{18.6 \times 20}{2}\right) \tan(24)(4 \times 0.46)(20) = 609.5 \text{ kN}$$

$$Q_{\text{all}} = \frac{1810.5 + 609.5}{4} = \mathbf{605 \text{ kN}}$$

$$11.11 \text{ Eq. (11.59): } q_{u(\text{design})} = \frac{q_{u(\text{lab})}}{5} = \frac{11,400}{5} = 2280 \text{ lb/in.}^2$$

$$N_\phi = \tan^2\left(45 + \frac{\phi'}{2}\right) = \tan^2\left(45 + \frac{36}{2}\right) = 3.852$$

$$\text{Eq. (11.61): } Q_{p(\text{all})} = \frac{q_{u(\text{design})}(N_\phi + 1)A_p}{\text{FS}}$$

For HP 14 × 102, $A_p = 30 \text{ in.}^2$

$$Q_{p(\text{all})} = \frac{(2280)(3.852 + 1)(30)}{(3)(1000)} = 110.6 \text{ kip}$$

$$11.12 \text{ Eq. (11.63): } s_{e(1)} = \frac{(Q_{wp} + \xi Q_{ws})L}{A_p E_p} = \frac{[70 + (0.57)(110)](50)}{(16 \times 16 \text{ in.}^2)(3 \times 10^3 \text{ kip/in.}^2)}$$

$$= 0.0086 \text{ ft} = 0.104 \text{ in.}$$

$$\text{Eq. (11.64): } s_{e(2)} = \frac{q_{wp} D}{E_s} (1 - \mu_s^2) I_{wp}$$

$$= \left[\left(\frac{70 \text{ kip}}{16 \times 16} \right) \left(\frac{16}{5 \text{ kip/in.}^2} \right) \right] (1 - 0.38^2)(0.85) = 0.636 \text{ in.}$$

$$\text{Eq. (11.66): } s_{e(3)} = \left(\frac{Q_{ws}}{pL} \right) \frac{D}{E_s} (1 - \mu_s^2) I_{ws}$$

$$I_{ws} = 2 + 0.35 \sqrt{\frac{50}{\left(\frac{16}{12}\right)}} = 4.13$$

$$s_{e(3)} = \left[\frac{110}{(4 \times 16)(50 \times 12)} \right] \left(\frac{16}{5} \right) (1 - 0.38^2)(4.13) = 0.032 \text{ in.}$$

$$s_e = 0.104 + 0.636 + 0.032 = 0.772 \text{ in.}$$

$$11.14 \quad I = \frac{1}{12}bh^3 = \frac{1}{12}(0.305)^4 = 0.0007 \text{ m}^4$$

$$E_p = 21 \times 10^6 \text{ kN/m}^2$$

$$\text{Eq. (11.80): } T = \sqrt[5]{\frac{E_p I_p}{n_h}} = \sqrt[5]{\frac{(21 \times 10^6)(0.0007)}{9200}} = 1.098 \text{ m}$$

$$\frac{L}{T} = \frac{30}{1.098} > 5. \text{ So, long pile. } M_g = 0. \text{ From Eq. (11.75)}$$

$$x_z(z) = A_x \frac{Q_g T^3}{E_p I_p}; \quad Q_g = \frac{x_z(z) E_p I_p}{A_x T^3}$$

$$\text{At } z = 0, x_z = 12 \text{ mm} = 0.012 \text{ m}; A_x = 2.435 \text{ (Table 11.13)}$$

$$Q_g = \frac{(0.012)(21 \times 10^6)(0.0007)}{(2.435)(1.098)^3} = 54.7 \text{ kN}$$

$$\text{Check for moment capacity: } M_{z(\max)} = F_y \frac{I_p}{d} = A_m Q_g T$$

$$Q_g = \frac{2F_y I_p}{d A_m T}$$

From Table 11.13, the maximum value of A_m is 0.772.

$$Q_g = \frac{(2)(21,000)(0.0007)}{(0.305)(0.772)(1.098)} = 113.7 \text{ kN}$$

Use $Q_p = 54.7 \text{ kN}$

$$11.16 \quad \text{Eq. (11.108): } Q_u = \frac{EH_E}{S+C}$$

$$H_E = 40 \text{ kip} \cdot \text{ft} = 40 \times 12 \text{ kip} \cdot \text{in.}; E = 0.85$$

$$Q_u = \frac{(0.85)(40 \times 12)}{\frac{1}{10} + 0.1} = 2040 \text{ kip}$$

$$Q_{\text{all}} = \frac{2040}{6} = 340 \text{ kip}$$

$$11.17 \quad Q_u = \frac{EW_R h}{S+C} \frac{W_R + n^2 W_p}{W_R + W_p}$$

$$W_p = \text{weight of (pile + cap)} = \frac{90 \times 100}{1000} + 2.4 = 11.4 \text{ kip}$$

$$Q_u = \left[\frac{(0.85)(40 \times 12)}{\frac{1}{10} + 0.1} \right] \left[\frac{12 + (0.35)^2 (11.4)}{12 + 11.4} \right] = 1167.9 \text{ kip}$$

$$Q_{\text{all}} = \frac{1167}{4} \approx 292 \text{ kip}$$

$$11.18 \quad Q_u = \frac{EH_E}{S + \sqrt{\frac{EH_E L}{2A_p E_p}}}; \sqrt{\frac{EH_E L}{2A_p E_p}} = \sqrt{\frac{(0.85)(40 \times 12)(90 \times 12)}{(2)(29.4)(30 \times 10^3)}} = 0.5$$

$$Q_u = \frac{(0.85)(36 \times 12)}{0.1 + 0.5} = 680 \text{ kip}$$

$$Q_{\text{all}} = \frac{680}{3} \approx 227 \text{ kip}$$

$$11.22 \text{ a. } \eta = \frac{2(n_1 + n_2 - 2)d + 4D}{\pi n_1 n_2}$$

$$d = 0.92 \text{ m}; D = 0.46 \text{ m. So}$$

$$\eta = \left[\frac{(2)(3 + 3 - 2)(0.92) + (4)(0.46)}{(\pi \times 0.46)(3)(3)} \right] (100) = \mathbf{70.7\%}$$

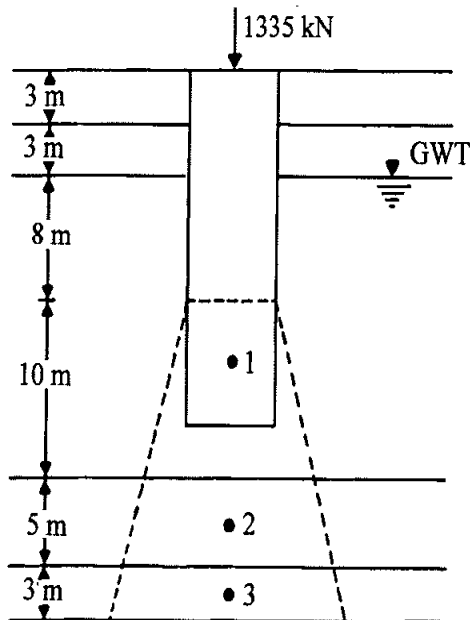
$$\begin{aligned} \text{b. } \eta &= 1 - \frac{D}{\pi d n_1 n_2} \left[n_1(n_2 - 1) + n_2(n_1 - 1) + \sqrt{2}(n_1 - 1)(n_2 - 1) \right] \\ &= 1 - \frac{0.46}{(\pi)(0.92)(3)(3)} \left[(3)(2) + (3)(2) + (\sqrt{2})(2)(2) \right] = 0.688 = \mathbf{68.8\%} \end{aligned}$$

$$11.23 \text{ a. } d = 1.2 \text{ m}$$

$$\eta = \left[\frac{(2)(3 + 3 - 2)(1.2) + (4)(0.46)}{(\pi \times 0.46)(3)(3)} \right] (100) = \mathbf{87.96\%}$$

$$\text{b. } \eta = 1 - \frac{0.46}{(\pi)(1.2)(3)(3)} \left[(3)(2) + (3)(2) + (\sqrt{2})(2)(2) \right] = 0.76 = \mathbf{76.06\%}$$

11.27 The pressure distribution diagram is shown.



Layer	σ'_o (kN / m ²)	$\Delta\sigma'$ (kN / m ²)	$\sigma'_o + \Delta\sigma'$ (kN / m ²)
1	$(15.72)(3) + (18.55 - 9.81)(3)$ $+ (13)(19.18 - 9.81)$ $= 47.16 + 26.22 + 121.81 = 195.19$	$\frac{1335}{(2.75+5)^2} = 22.23$	217.42
2	$195.19 + (5)(19.18 - 9.81)$ $+ (2.5)(18.08 - 9.81) = 262.72$	$\frac{1335}{(2.75+12.5)^2} = 5.74$	268.46
3	$262.72 + (2.5)(18.08 - 9.81)$ $+ (1.5)(19.5 - 9.81) = 297.93$	$\frac{1335}{(2.75+16.5)^2} = 3.6$	301.53

$$\Delta s_{c(1)} = \frac{(0.8)(10)}{1+0.8} \log\left(\frac{217.42}{195.19}\right) = 0.208 \text{ m}$$

$$\Delta s_{c(2)} = \frac{(0.31)(5)}{1+1} \log\left(\frac{268.46}{262.72}\right) = 0.0073 \text{ m}$$

$$\Delta s_{c(3)} = \frac{(0.26)(3)}{1+0.7} \log\left(\frac{301.53}{297.93}\right) = 0.0024 \text{ m}$$

$$\Sigma \Delta s_c \approx 217.7 \text{ mm}$$

$$12.7 \quad a. \quad Q_p = A_p c_{u(2)} N_c^*$$

$$N_c^* = 1.33[\ln(I_r) + 1]$$

$$\frac{c_{u(2)}}{p_a} = \frac{1800}{2000} = 0.9$$

$$\text{From Eq. (12.41) and Figure 12.15: } I_r = \frac{E_s}{3c_{u(2)}} = 237.5$$

$$N_c^* = 1.33[\ln(237.5) + 1] = 8.61$$

$$Q_p = \frac{\pi}{4} (5)^2 (1800) (8.61) = 304,302 \text{ lb} = \mathbf{304 \text{ kip}}$$

$$b. \quad Q_s = \alpha^* c_{u(1)} p L_1 + \alpha^* c_{u(2)} p L_2 = (0.4)(\pi \times 5)[(1000)(20) + (1800)(15)]$$

$$= 295,310 \text{ lb} = \mathbf{295.3 \text{ kip}}$$

$$c. \quad Q_w = \frac{304.3 + 295.3}{4} \approx \mathbf{149.9 \text{ kip}}$$

12.11 a. $\Delta L_1 = 20 - 5 = 15 \text{ ft}; c_{u(1)} = 1000 \text{ lb / ft}^2$

$$\Delta L_2 = 15 - 5 = 10 \text{ ft}; c_{u(2)} = 1800 \text{ lb / ft}^2$$

$$\Sigma f_i p \Delta L_i = (0.55)[(1000)(\pi \times 5)(15) + (2800)(\pi \times 5)(10)] = 472,593 \text{ lb} \approx 371.5 \text{ kip}$$

$$q_p = 6c_{ub} \left(1 + 0.2 \frac{L}{D_b} \right) = (6)(1800) \left[1 + (0.2) \left(\frac{35}{5} \right) \right] = 25,920 \text{ lb / ft}^2$$

$$\text{Check: } q_b = 9c_{ub} = (9)(1800) = 16,200 \text{ lb / ft}^2$$

Use $q_b = 16,200 \text{ lb / ft}^2$

$$Q_u = 371.5 + (16.2) \left(\frac{\pi}{4} \times 5^2 \right) = 689.6 \text{ kip}$$

b. $\frac{\text{allowable settlement}}{D_s} = \frac{1 \text{ in.}}{(5)(12)} = 1.66\%$

From Figure 12.16, normalized side load $\approx (0.87)(\text{ultimate side load})$

$$\frac{\text{allowable settlement}}{D_b} = \frac{1}{(5)(12)} = 1.67\%$$

From Figure 12.17, normalized end bearing $\approx (0.77)(\text{ultimate end bearing})$

$$Q = (0.87)(371.5) + (0.77)(16.2) \left(\frac{\pi}{4} \times 5^2 \right) = 568.1 \text{ kip}$$