57:019:AAA Mechanics of Deformable Bodies
Review Material

Overview:

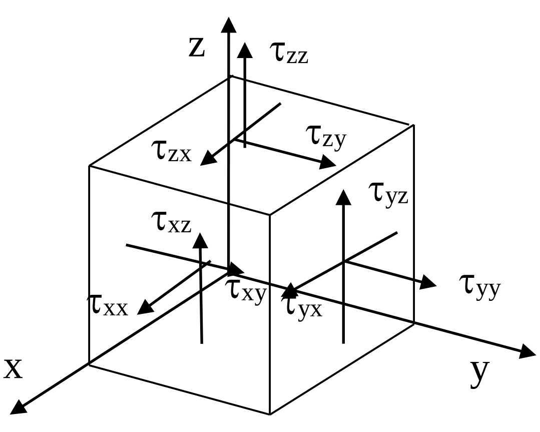
A. Key Ideas of Stress
B. Key Ideas of Strain
C. Hooke's Law for Linear Isotropic Elastic Solids
D. Axial Loads and Deformation
E. Torsion and Twist
F. Bending Behaviors
    1. V, M diagrams
    2. Flexure Formulae
    3. Composite Sections
    4. Shear Stress & Shear Flow
    5. Deflections
G. Euler Buckling
H. Energy Methods
I. Stress Transformations

A. Key Ideas of Stress

Stress is defined as <u>force</u> per unit <u>area</u>. When working with stresses, we need to be very specific about the <u>directionality</u> of the forces and the <u>directionality</u> of the area on which the forces are acting. This is done by representing stress as a rank-2 tensor as follows:

$$\boldsymbol{\tau} = \begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix} = \begin{bmatrix} \sigma_{x} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{y} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{z} \end{bmatrix}$$

The diagonal entries in the stress tensor are said to be **normal stresses** because the associated force component acts normal to the plane.



The off-diagonal entries in the stress tensor are called **shear stresses**, because the associated force components act parallel to the plane.

It is very straightforward to demonstrate that the stress tensor is symmetric, or specifically that:

$$\tau_{xy} = \tau_{yx}$$
$$\tau_{yz} = \tau_{zy}$$

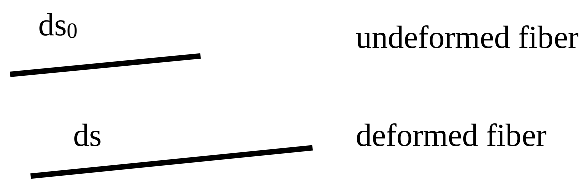
$$\tau_{zx} = \tau_{xz}$$

B. Key Ideas of Strain

1. Normal strains

Normal strain measures the change in length of an infinitesimal “fiber” of material relative to the original length of that fiber.

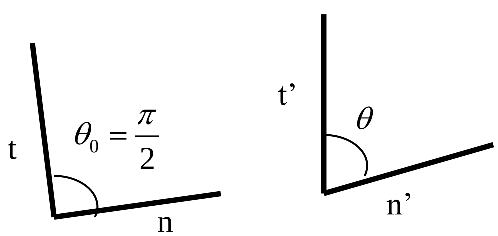
$$\varepsilon = \frac{ds - ds_0}{ds_0}$$



$$\varepsilon < 0 \Rightarrow \text{reduction of length}$$
$$\varepsilon = 0 \Rightarrow \text{no change of length}$$
$$\varepsilon > 0 \Rightarrow \text{increase of length}$$

2. Shear Strains

Shear strain represents the negative change in angle (radians) during deformation between two **infinitesimal fibers** that were initially perpendicular.



shear strain: $\gamma_{nt} = \frac{\pi}{2} - \theta$

undeformed deformed

To describe the change in length of fibers aligned with Cartesian reference axes, and to quantify change in angles between these fibers, a second rank strain tensor is used:

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix}$$

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x}; \quad \varepsilon_{yy} = \frac{\partial u_y}{\partial y}; \quad \varepsilon_{zz} = \frac{\partial u_z}{\partial z};$$

$$\varepsilon_{xy} = \varepsilon_{yx} = \tfrac{1}{2}\gamma_{xy} = \tfrac{1}{2}\gamma_{yx} = \tfrac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right);$$

$$\varepsilon_{xz} = \varepsilon_{zx} = \tfrac{1}{2}\gamma_{xz} = \tfrac{1}{2}\gamma_{zx} = \tfrac{1}{2}\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right);$$

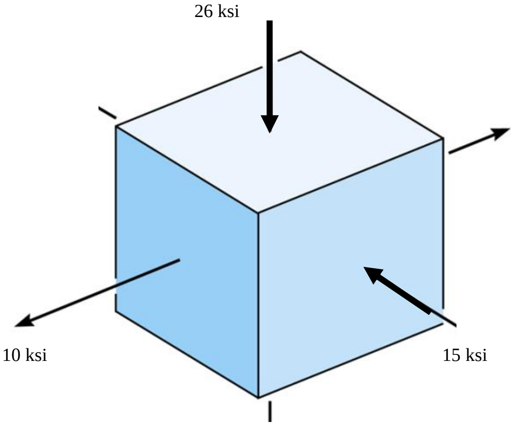
$$\varepsilon_{yz} = \varepsilon_{zy} = \tfrac{1}{2}\gamma_{yz} = \tfrac{1}{2}\gamma_{zy} = \tfrac{1}{2}\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right);$$

C. Hooke's Law for Isotropic, Elastic Material

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{zx}\end{bmatrix} = \frac{E}{1+\nu}\begin{bmatrix}\left(\frac{1-\nu}{1-2\nu}\right) & \left(\frac{\nu}{1-2\nu}\right) & \left(\frac{\nu}{1-2\nu}\right) & 0 & 0 & 0\\ \left(\frac{\nu}{1-2\nu}\right) & \left(\frac{1-\nu}{1-2\nu}\right) & \left(\frac{\nu}{1-2\nu}\right) & 0 & 0 & 0\\ \left(\frac{\nu}{1-2\nu}\right) & \left(\frac{\nu}{1-2\nu}\right) & \left(\frac{1-\nu}{1-2\nu}\right) & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}\end{bmatrix}\cdot\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{zx}\end{bmatrix}$$

$$\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{zx}\end{bmatrix} = \frac{1}{E}\begin{bmatrix}1 & -\nu & -\nu & 0 & 0 & 0\\ -\nu & 1 & -\nu & 0 & 0 & 0\\ -\nu & -\nu & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 2(1+\nu) & 0 & 0\\ 0 & 0 & 0 & 0 & 2(1+\nu) & 0\\ 0 & 0 & 0 & 0 & 0 & 2(1+\nu)\end{bmatrix}\cdot\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{zx}\end{bmatrix}$$

Example:
The principal stresses at a point are shown in the figure. If the material is aluminum for which $E_{al}=1*10^4$ ksi and $\nu_{al}=0.33$, determine the principal strains.



$$\sigma_x = 10ksi; \sigma_y = -15ksi; \sigma_z = -26ksi$$

$$\varepsilon_{xx} = \frac{1}{10^4 ksi}\left[10 - \tfrac{1}{3}(-15) - \tfrac{1}{3}(-26)\right] = +.00237$$

$$\varepsilon_{yy} = \frac{1}{10^4 ksi}\left[-\tfrac{1}{3}(10) + (-15) - \tfrac{1}{3}(-26)\right] = -.000967$$

$$\varepsilon_{zz} = \frac{1}{10^4 ksi}\left[-\tfrac{1}{3}(10) - \tfrac{1}{3}(-15) + (-26)\right] = -.00277$$

D. Axial Strains and Deformations

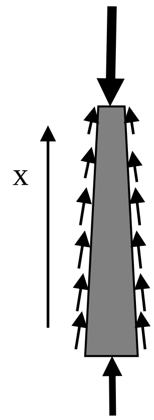
<u>Constant force, area</u>

$$\sigma_{axial} = F/A$$

$$\varepsilon_{axial} = \frac{\sigma_{axial}}{E} = \frac{F}{AE} = \frac{\Delta L}{L_0}$$

$$\therefore \Delta L = \frac{FL_0}{AE}$$

<u>Variable force, area</u>



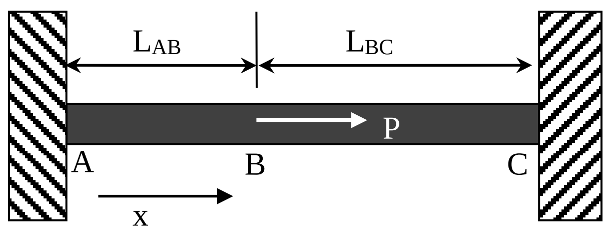
$$\sigma(x) = \frac{F(x)}{A(x)}$$

$$\varepsilon(x) = \frac{\sigma(x)}{E(x)} = \frac{F(x)}{A(x)E(x)}$$

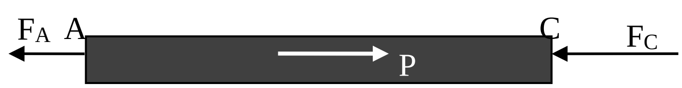
$$\Delta L = \int_0^{L_0} \varepsilon(x)dx = \int_0^{L_0} \frac{F(x)}{A(x)E(x)}dx$$

Indeterminate Axially Loaded Members

In this illustrative example, there is only one relevant equation of equilibrium $\left(\sum F_x = 0\right)$ but there are two support reactions at A and C to solve for. Thus the system is statically indeterminate.

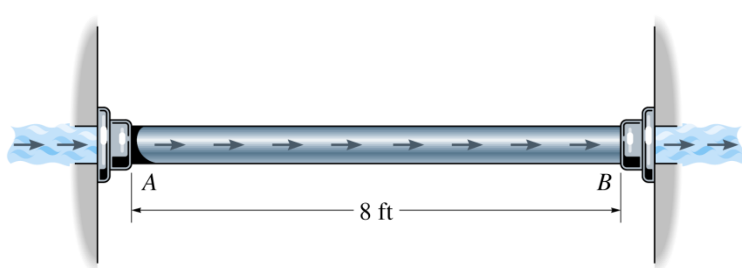


To solve for both support reactions, we need an additional condition. Here, since the supports at both A and C are taken to be rigid, we can say that $\delta_{C/A}=0$ or that overall the axial member does not change its length.



$$\sum F_x = 0 \Rightarrow P = F_A + F_C$$

Example: The bronze 86100 pipe has an inner radius of 0.5 in. and a wall thickness of 0.2 in. If the gas flowing through it changes the temperature of the pipe uniformly from T_A=200°F at A to T_B=60°F at B, determine the axial force it exerts on the walls. The pipe was fitted between the walls when T=60°F. E=15,000ksi and $\alpha = 9.6 \cdot 10^{-6} / {}^\circ F$



Solution:
From a free-body diagram, there are two equal and compressive wall forces F_A=F_B acting on the pipe that tend to compress it. Alternatively, the temperature increase in the pipe generates thermal strains that causes expansion. The resulting axial strain is as follows:

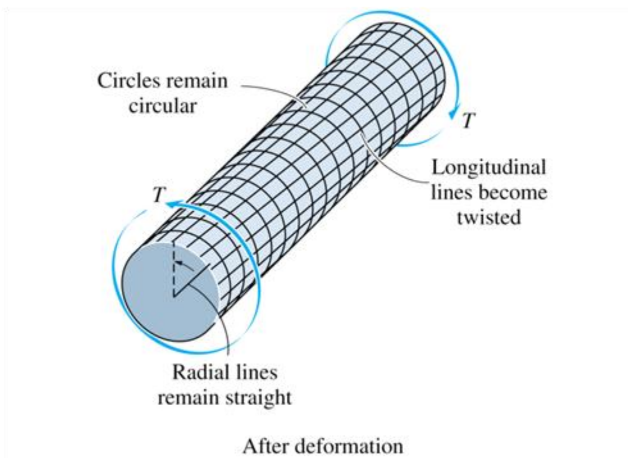
$$\varepsilon(x) = \alpha\Delta T(x) - \frac{F_B}{AE} \text{ where } \Delta T(x) = \Delta T_o\left(1 - \frac{x}{L}\right) \text{and } \Delta T_o = 140^\circ F$$

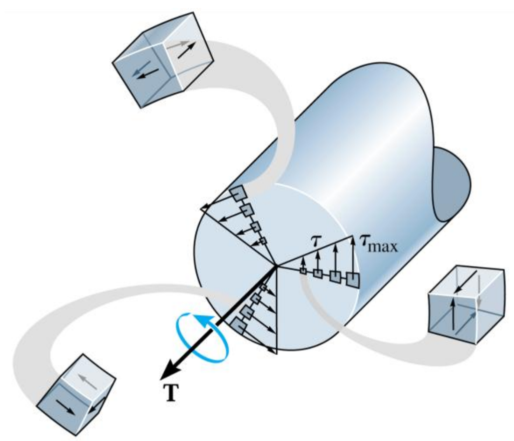
Because the two walls are rigid can be no change in length of the pipe.

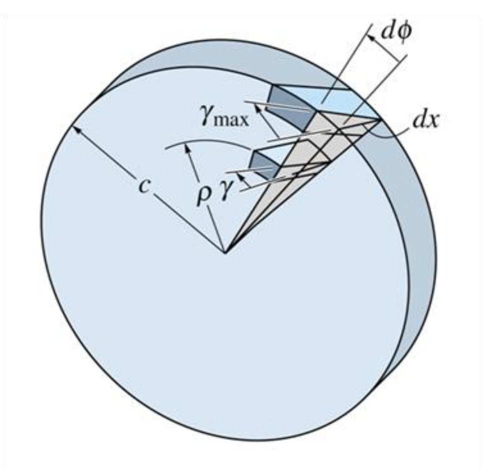
$$\Delta L = \int_0^L \varepsilon(x)dx = 0 = \int_0^L \left[\alpha\Delta T_o\left(1 - \frac{x}{L}\right) - \frac{F_B}{AE}\right]dx$$

Solving for F_B gives: $\boxed{F_B = \frac{\alpha\Delta T_o AE}{2} = 7.6kip = F_A}$

E. Torsion and Twist







The shear strain for the material increases linearly with ρ, i.e., $\gamma = (\rho/c)\gamma_{max}$

$$\gamma = \rho\frac{d\phi}{dx} = \frac{\rho}{c}\gamma_{\max}$$

where: γ is the material's shear strain
ρ is the distance of a material point from the centroidal axis
c is the radius of the circular cross-section
ϕ is the angle of twist in the member
x is the coordinate variable along the member's centroidal axis

Integrating the moment of shear stresses over a circular cross-section about the longitudinal axial of the member we obtain the relation between shear stresses and torque:

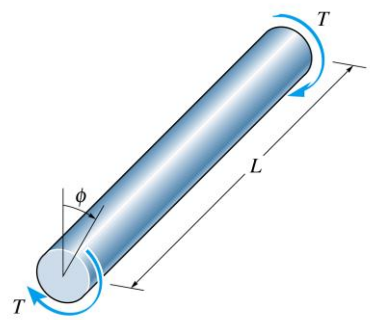
$$\begin{aligned} T &= \int_A \rho\tau \, dA \\ &= \int_0^c 2\pi\tau\rho^2 d\rho \\ &= \frac{\tau_{\max}}{c}\frac{\pi c^4}{2} \end{aligned}$$

$$\begin{aligned} J &= \int_A \rho^2 dA \\ &= \int_0^c 2\pi\rho^3 d\rho \\ &= \frac{\pi c^4}{2} \end{aligned}$$

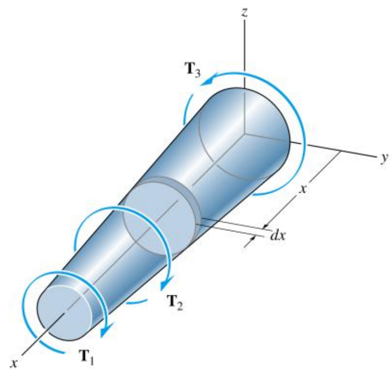
$$\tau_{\max} = \frac{Tc}{J}$$

$$\tau = \frac{T\rho}{J}$$

$$\frac{d\phi}{dx} = \frac{T}{GJ}$$



$$\phi = \frac{TL}{JG}$$

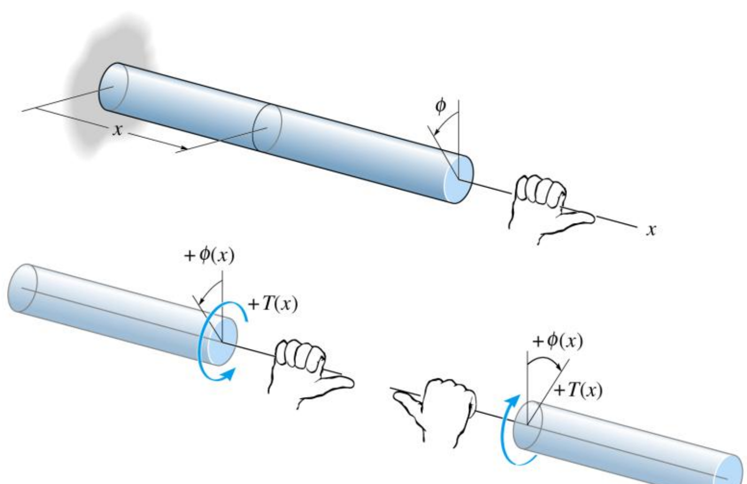


(a)

$$\phi = \int_0^L \frac{T(x)dx}{J(x)G}$$

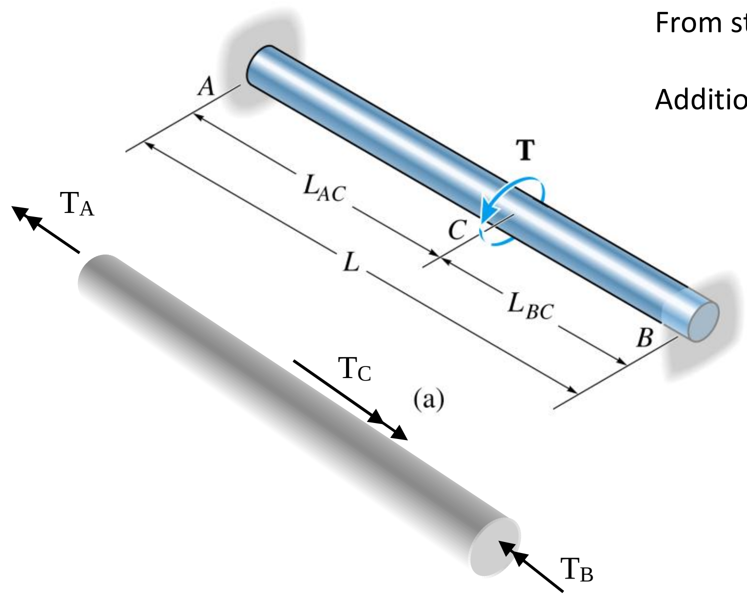
Sign Convention:

- **<u>Internal torques and their associated angle change are positive when the resultant vector points away from the cut face on which it acts.</u>**



Positive sign convention for T and ϕ

Indeterminate Torsion Example:



From statics: $T_A+T_B=T_C$

Additional constraint: $\phi_{AB}=0$

$$\begin{aligned}
\phi_{AB} &= 0 \\
&= \int_A^C \frac{T_{AC}}{JG}dx + \int_C^B \frac{T_{CB}}{JG}dx \\
&= \frac{T_A L_{AC}}{JG} - \frac{(T_C - T_A)L_{BC}}{JG}
\end{aligned}$$

$$T_A\left(L_{AC}+L_{CB}\right)=T_C L_{BC}$$

$$T_A = \frac{T_C L_{BC}}{L_{AB}}; \qquad T_B = \frac{T_C L_{AC}}{L_{AB}}$$

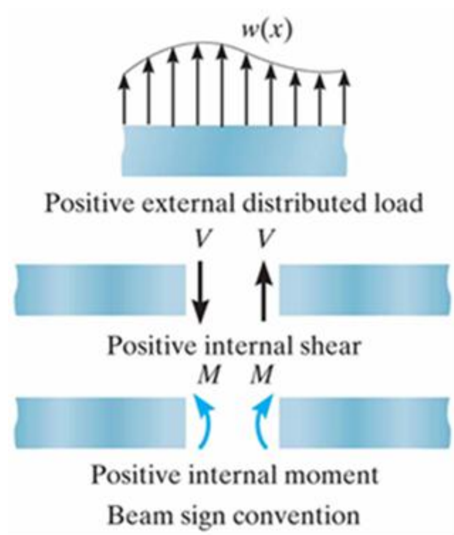
F. Bending Behaviors

Shear and Bending Moment Diagrams

Bending occurs in members that experience loads perpendicular to the longitudinal axis.

$$\frac{dV}{dx} = w(x) \Rightarrow \left(\Delta V\right)_{AB} = \int_A^B w(x)dx$$

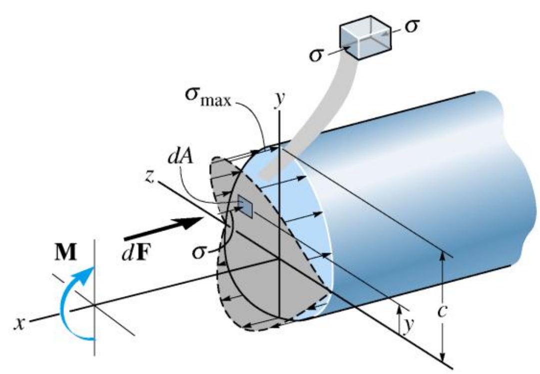
$$\frac{dM}{dx} = V(x) \Rightarrow \left(\Delta M\right)_{AB} = \int_A^B V(x)dx$$



The flexure formula:

One-way bending: $\sigma = \frac{-My}{I}; \qquad \sigma_{\max} = \frac{-Mc}{I}$

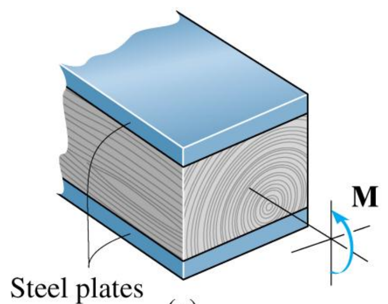
Biaxial Bending: $\sigma(y,z) = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$



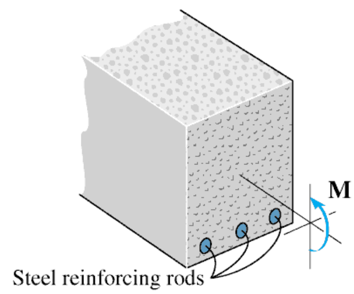
Bending stress variation

Composite sections:

Beams are often made of different materials in order to efficiently carry a load.



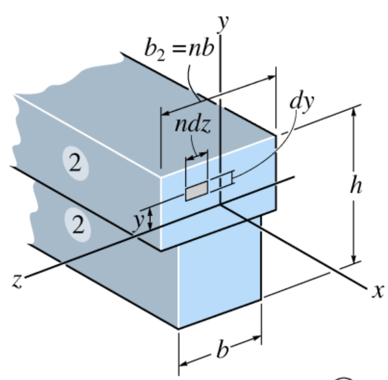
(a)



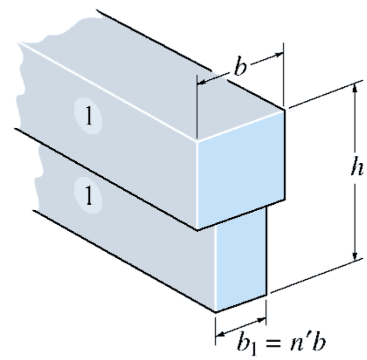
(b)

- What is the elastic bending stress distribution on the composite cross-section?

The cross section of the beam must be transformed into a single material if the flexure formula (which is based on homogenous materials) is to be used to compute the bending stress. A transformation factor "$n=E_1/E_2$" is used for this purpose. Once the section is transformed, I* is computed.



Beam transformed to material ②
(e)



Beam transformed to material ①
(f)

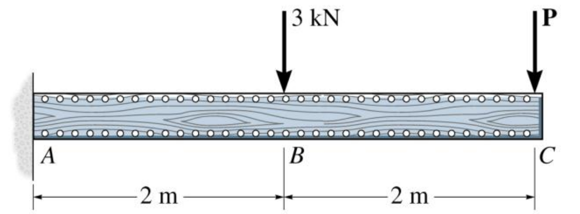
To calculate stresses on the cross-section:

- In the original material: $\sigma = \dfrac{-M\ y}{I^*}$. In the transformed material: $\sigma = \dfrac{-nM\ y}{I^*}$

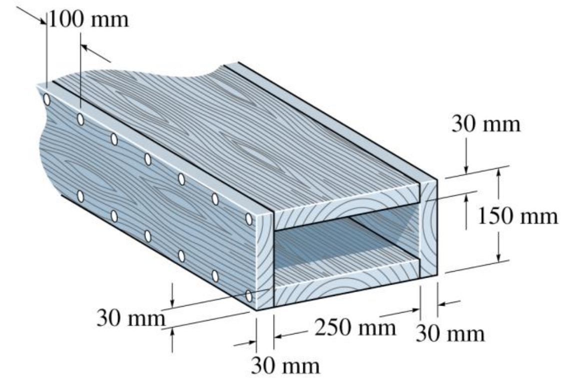
The transverse shear formula: $\tau = \dfrac{VQ}{It}$ where $Q = \int_{A'} y\, dA'$

Shear Flow: $q = \dfrac{V*Q}{I}$

- Shear Flow is denoted by "q" and denotes the shear force per unit length transmitted along a specified longitudinal section.
- Usually, we are interested in the shear flow along sections where different members are joined.
- Here, Q is the moment about the NA of the cross-sectional area connected to the remainder of the section along the longitudinal section of interest.



Example: The cantilever beam is subjected to the loading shown, where P=7kN. Determine the average shear stress developed in the nails within region AB of the beam. The nails are located on each side of the beam and are spaced 100mm apart. Each nail has a diameter of 5mm.



Solution:

$V_{\max} = 10kN$ from A to B

$$I = \frac{(310)(150)^3}{12} - \frac{(250)(90)^3}{12} = 7.2\cdot 10^7\, mm^4$$

$$q = \frac{VQ}{I}$$ take the Q for the top board

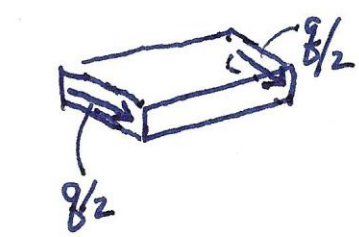
$$Q = \bar{y}'A' = (30)(250)(60) = 4.5\cdot 10^5\, mm^3$$

$$q = \frac{(10kN)\left(4.5\cdot 10^{-4} m^3\right)}{7.2\cdot 10^{-5} m^4} = 62.5kN/m$$

Half of this shear flow comes along each edge of the board

Shear flow along one edge of board is 31.25kN/m

Shear force V_{nail} in a nail is shear flow along edge * spacing:



$$\boxed{\begin{aligned} V_{nail} &= (31.25kN/m)(0.1m) = 3.125kN \\ \tau_{ave} &= \frac{V_{nail}}{A_{nail}} = \frac{3.125kN}{\frac{\pi}{4}(.005m)^2} = 159MPa \end{aligned}}$$

Computing Deflections in Beams:

1. Determinate Beams:
   a. Starting with M(x), integrate to find θ(x).
   b. Integrate θ(x) to obtain v(x).

2. Indeterminate Beams: (Use kinematic constraints from excess support reactions to solve for the coefficients of integration.)

   a. Starting with w(x), integrate to obtain V(x)
   b. Integrate V(x) to obtain M(x)
   c. Integrate M(x) to obtain θ(x)
   d. Integrate θ(x) to obtain v(x)

$$\frac{dV}{dx} = w(x) = EI\frac{d^4v}{dx^4}$$

$$\frac{dM}{dx} = V(x) = EI\frac{d^3v}{dx^3}$$

$$\frac{d\theta}{dx} = \frac{M(x)}{EI} = \frac{d^2v}{dx^2} = \kappa(x)$$

$$\frac{dv}{dx} = \theta(x)$$

Example: Compute the slope of the elastic curve at A and B, and the deflection at C. EI is constant. **Solution:**

$$V(x) = -wx + c_1$$

$$= -wx + \frac{wL}{2} \text{ since } V(0) = \frac{wL}{2}$$

$$M(x) = -\frac{wx^2}{2} + \frac{wLx}{2} + c_2$$

$$= -\frac{wx^2}{2} + \frac{wLx}{2} \text{ since } M(0)=0$$

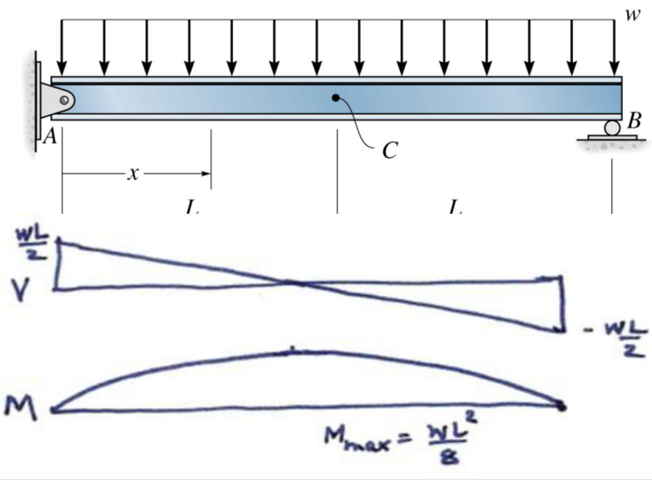
$$EI\theta(x) = -\frac{wx^3}{6} + \frac{wLx^2}{4} + c_3$$

$$EIv(x) = -\frac{wx^4}{24} + \frac{wLx^3}{12} + c_3 x + c_4$$

$$= -\frac{wx^4}{24} + \frac{wLx^3}{12} + c_3 x \text{ since } v(0)=0$$

$$EIv(L)=0 \;\rightarrow c_3 = \frac{-wL^3}{24}$$

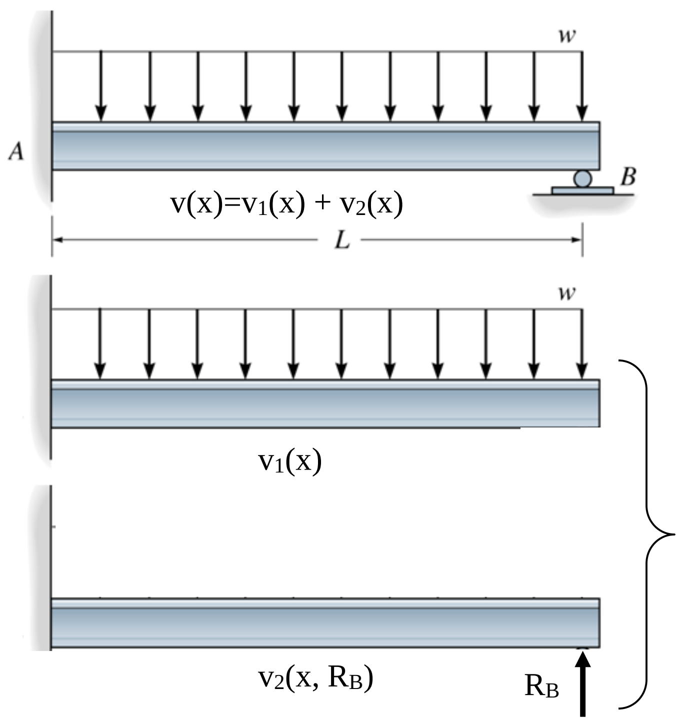
$$\therefore v(x) = \frac{-wx}{24EI}\left[x^3 - 2Lx^2 - L^3\right]$$



$$\boxed{\begin{aligned} v_C &= v\left(\frac{L}{2}\right) = \frac{-5wL^4}{384EI} \\ \theta_A &= \theta(0) = \frac{-wL^3}{24EI} \\ \theta_B &= \theta(L) = \frac{wL^3}{24EI} \end{aligned}}$$

Principle of Superposition

- Statically indeterminate beams are those that have more supports than there are relevant equilibrium equations. For such beams, one cannot just use the equations of static equilibrium to solve for the support reactions.
- Solving for the displacements, slopes, shears, and moments in statically



Solve for R_B such that:
$v_1(L) + v_2(L) = 0$

indeterminate beams is actually very straightforward, because whenever a support condition exists for the beam, there is a corresponding kinematic constraint on the beam.

- Usage of superposition is one way to solve statically indeterminate beam problems.

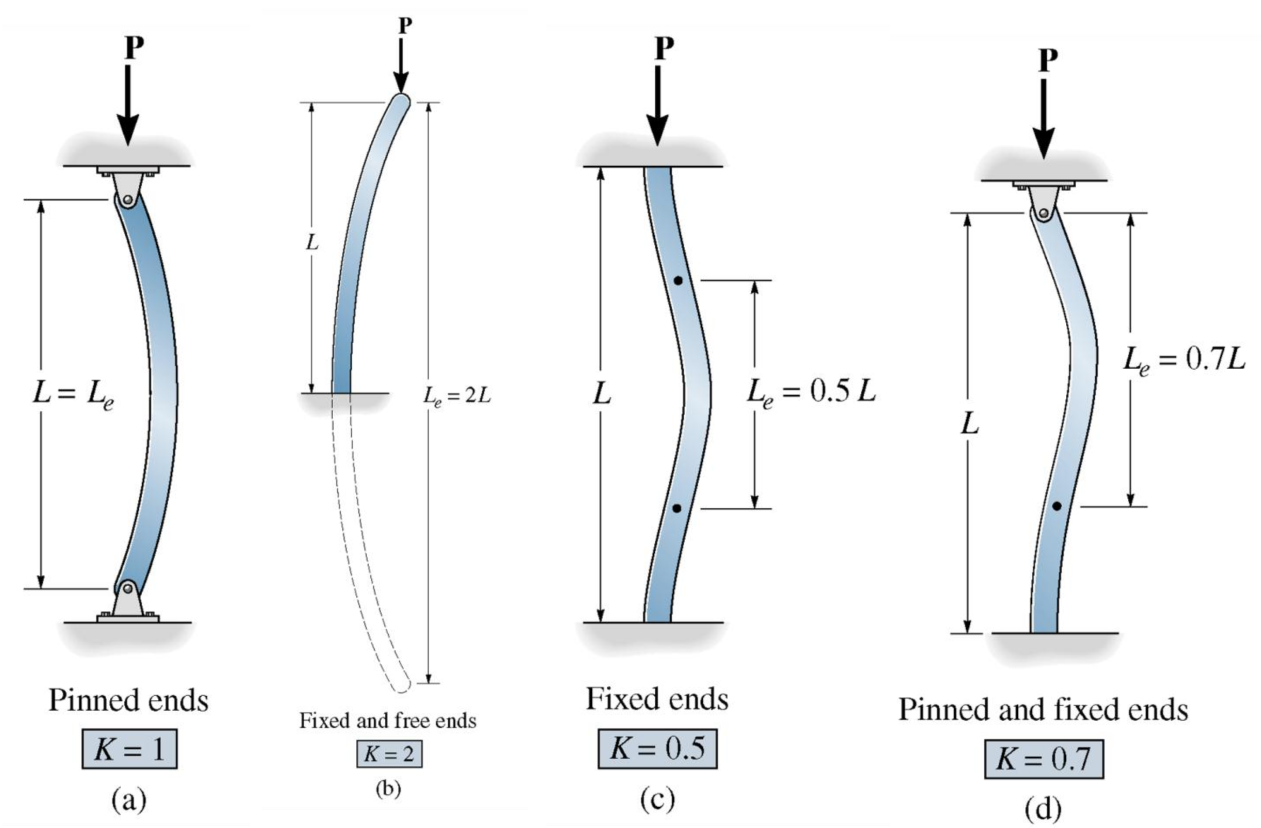
G. Euler Buckling Loads with different types of supports

Buckling is primarily a concern or issue in long slender members subjected to axial compression.

The critical axial compression load that will cause buckling is computed as $P_{cr} = \frac{EI\pi^2}{(KL)^2}$

where: E is the Young's modulus, I is the minimum moment of inertia of the cross-section, and L is the length of the member.

Depending upon the manner in which the compression member is restrained at its ends, the effective length factor K will change as shown in the diagrams below.



H. Energy Methods

1. Elastic Strain Energies

a. Axial Loading

$$U = \int u\, dV = \int \frac{\sigma_x^{\ 2}}{2E}\, dV = \int \frac{\sigma_x^{\ 2}}{2E} A dx = \int \frac{N^2}{2A^2 E} A dx = \int \frac{N^2}{2AE} dx$$

b. Bending

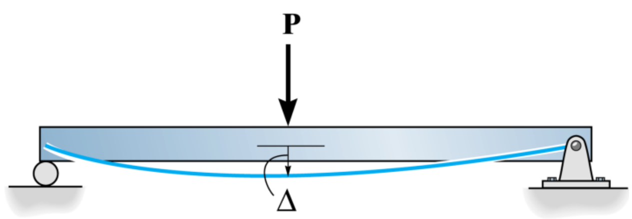
$$U = \int u\, dV = \int \frac{(M(x)y)^2}{2I^2 E} dV = \int \frac{M^2(x)}{2I^2 E} \int y^2 dA\, dx = \int \frac{M^2(x)}{2IE} dx$$

c. Torsional Deformation

$$U = \int u\, dV = \int \frac{\tau^2}{2G} dV = \int \frac{T^2 \rho^2}{2GJ^2} A dx = \int \frac{T^2}{2GJ^2} \int \rho^2 dA\, dx = \int \frac{T^2}{2GJ} dx$$

2. Conservation of Energy (quasi-static systems)

$\dfrac{P \cdot \mathbf{\Delta}}{2} = U$ Knowing U and P, can solve for Δ $\quad \Delta = \dfrac{2U}{P} = \dfrac{2}{P}\displaystyle\int_0^L \frac{M^2}{2EI} dx$



Using Conservation of Energy, displacement can only be computed at the same point and in the same direction as the applied load.

3. Principle of Virtual Work (More General)

- A mechanical system will be subjected to a real set of loads **F**, and this will result in real strains $\boldsymbol{\varepsilon}$ in the material that comprises the system.
- Imagine that before the system is subjected to the real loads **F** it is first subjected to an infinitesimally small virtual load $\mathbf{f}_v$ at the point in the system where we desire to know the

displacement, in the direction we wish to quantify the displacement. This virtual load will give rise to equilibrium virtual stresses $\boldsymbol{\sigma}_v$ and virtual strains $\boldsymbol{\varepsilon}_v$ in the system.

- Next, the real loads **F** are applied to the system resulting in real displacements $\boldsymbol{\delta}$ throughout the system as well as real equilibrium stresses $\boldsymbol{\sigma}$ and strains $\boldsymbol{\varepsilon}$ in the system.
- Externally, the work done by the virtual force as the structure undergoes real displacements is: $W_v = \mathbf{f}_v \cdot \boldsymbol{\delta}$
- Internally, the virtual strain energy associated with the real strains and the virtual stresses is $U_v = \int_V (\boldsymbol{\sigma}_v : \boldsymbol{\varepsilon})\, dV$
- Equating the external virtual work with the internal virtual strain energy we get:

$$W_v = \mathbf{f}_v \cdot \boldsymbol{\delta} = \int_V (\boldsymbol{\sigma}_v : \boldsymbol{\varepsilon})\, dV = U_v$$

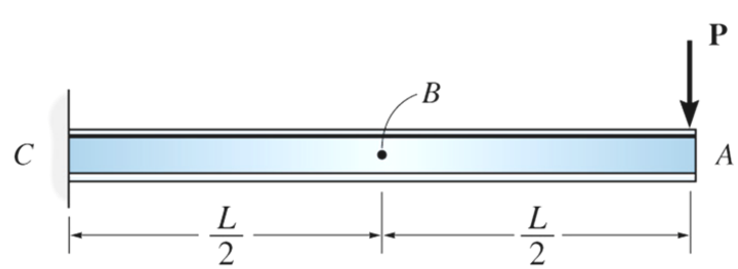
- The real displacement magnitude at the location of the virtual force and parallel to the direction of the virtual force is:

$$\frac{\mathbf{f}_v \cdot \boldsymbol{\delta}}{\|\mathbf{f}_v\|} = \frac{1}{\|\mathbf{f}_v\|} \int_V (\boldsymbol{\sigma}_v : \boldsymbol{\varepsilon})\, dV$$

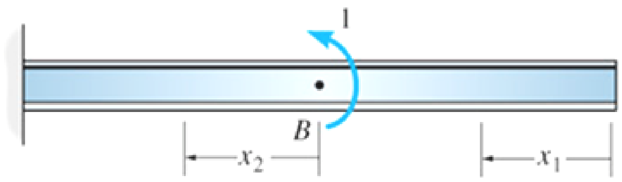
Deformation caused by	Strain energy	Internal virtual work
Axial load N	$\int_0^L \frac{N^2}{2EA}\,dx$	$\int_0^L \frac{nN}{EA}\,dx$
Shear V	$\int_0^L \frac{f_s V^2}{2GA}\,dx$	$\int_0^L \frac{f_s vV}{GA}\,dx$
Bending moment M	$\int_0^L \frac{M^2}{2EI}\,dx$	$\int_0^L \frac{mM}{EI}\,dx$
Torsional moment T	$\int_0^L \frac{T^2}{2GJ}\,dx$	$\int_0^L \frac{tT}{GJ}\,dx$

Example:

- Consider the cantilever beam shown. If we wanted to compute the vertical displacement at A, we could use conservation of energy.



- But if we wanted to compute the slope at B, the principle of virtual work might be more direct.



- First, we apply a virtual moment at B as shown below.

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- The resulting virtual moment distribution $m_v(x)$ in the beam from the virtual moment at B would be as follows:

$$m_v(x) = \begin{Bmatrix} 1 & 0 \le x \le L/2 \\ 0 & x > L/2 \end{Bmatrix}$$

- ***The virtual bending stresses in the beam would be as follows:***

$$\sigma_v(x,y) = \frac{-m_v y}{I} = \begin{Bmatrix} \dfrac{-y}{I} & 0 \le x \le L/2 \\ 0 & x > L/2 \end{Bmatrix}$$

- When the real load P is applied to the tip of the beam, the real moment distribution in the beam is:

$$M(x) = -PL\left(1 - \frac{x}{L}\right)$$

- The resulting real bending strains in the beam are $\varepsilon(x,y) = \dfrac{-M(x)y}{EI}$:

- The virtual strain energy in the beam would be as follows:

$$\begin{aligned} U_v &= \int_V \frac{m_v(x)M(x)y^2}{EI^2} dV = \int_0^L \frac{m_v(x)M(x)}{EI^2} \int_A y^2 dA\, dx = \int_0^L \frac{m_v(x)M(x)}{EI} dx \\ &= \int_0^{L/2} \frac{1 \cdot M(x)}{EI} dx = \frac{-PL}{EI} \int_0^{L/2} \left(1 - \frac{x}{L}\right) dx = \frac{-PL}{EI}\left[\frac{L}{2} - \frac{L}{8}\right] \\ &= \frac{-3PL^2}{8EI} \end{aligned}$$

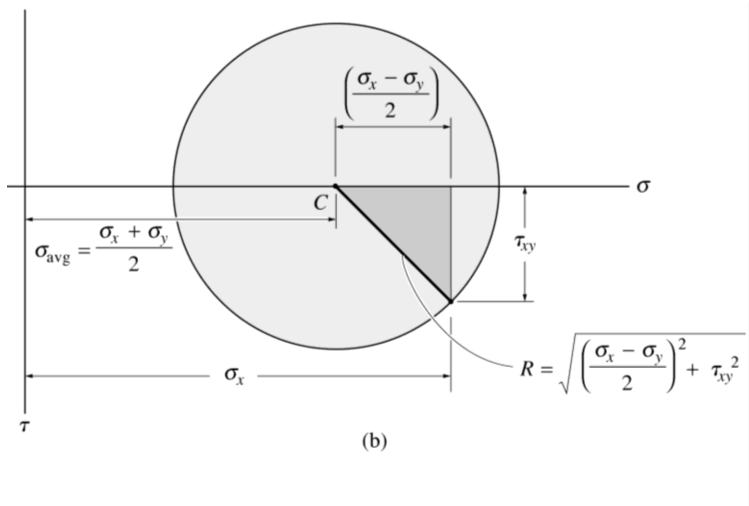
- ***The external virtual work is:*** $W_v = 1 \cdot \theta_B$

- ***Equating the external virtual work and virtual strain energy gives:*** $\theta_B = \dfrac{-3PL^2}{8EI}$

I. STRESS TRANSFORMATIONS

1. Plane Stress

- A state of plane stress at a point is defined by specifying the normal and shear stress components on two perpendicular planes: For example: σ_x, σ_y, τ_{xy}
- All combinations of normal and shear on other planes passing through the same point plot on a circle (Mohr's circle) in σ-τ space.
- The radius R of the circle and the center C on the σ-axis are given by the following relations:



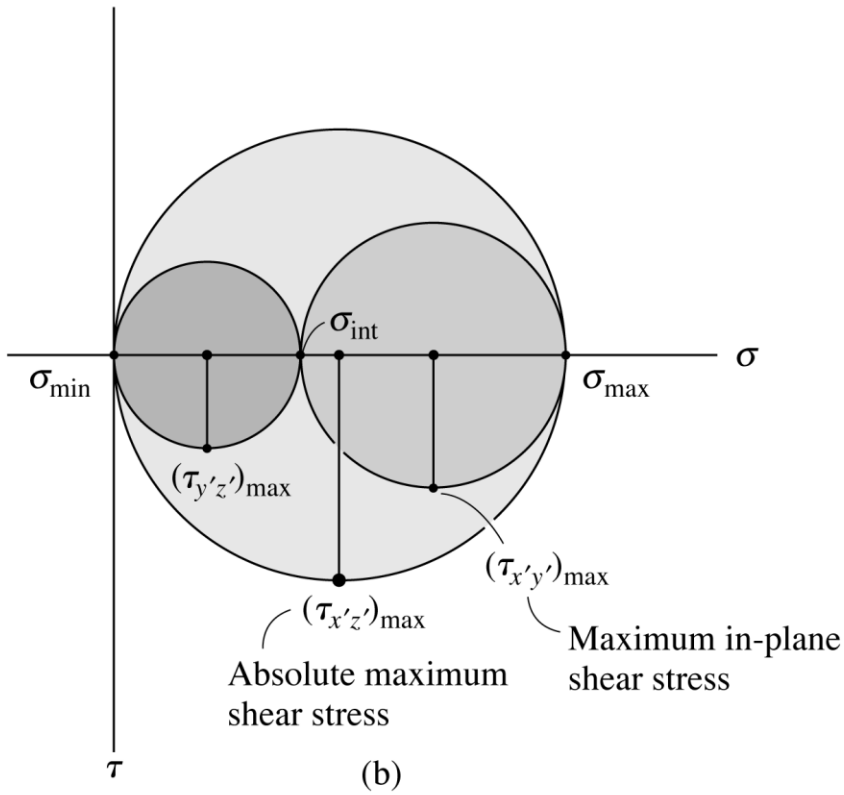
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad ; \qquad C = \sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

- Once the Mohrs Circle is known, the in-plane principal stresses are easily found:
  - Major principal stress: $\sigma_1 = C + R$
  - Minor principal stress: $\sigma_3 = C - R$
- In plane maximum shear stress: $\tau_{\max} = R$

2. Triaxial States of Stress:

First find all three principal stresses:

- σ_1 = major principal stress
- σ_2 = intermediate principal stress
- σ_3 = minor principal stress
- Then the absolute maximum shear stress is: $\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2}$
- The normal stress that corresponds with the absolute max shear stress:
  $\sigma_{ave} = \frac{\sigma_1 + \sigma_3}{2}$


σ_{int}
σ
σ_{min}
σ_{max}
$(\tau_{y'z'})_{max}$
$(\tau_{x'y'})_{max}$
$(\tau_{x'z'})_{max}$
Maximum in-plane
shear stress
Absolute maximum
shear stress
τ

(b)