

**ENGR:2750 Mechanics of Deformable Bodies**  
**College of Engineering, The University of Iowa**  
**Summer 2014, Quiz #1**

SOLUTION

**Question #1.** For the structural system shown, calculate the force and moment resultants acting on the cross-section a-a.

Use the FBD (free-body diagram) of the whole struct. to find support reactions

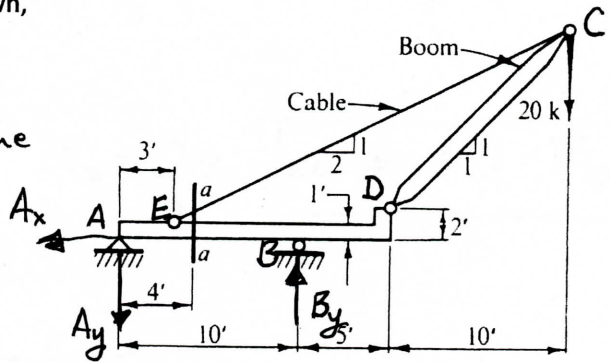
$$\sum M_A = 0 = B_y \cdot 10' - (20 \text{ kip})(25')$$

$$B_y = 50 \text{ kip} \uparrow$$

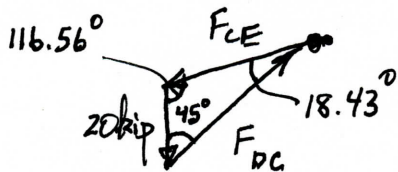
$$\sum F_y = 0 = -A_y + B_y - 20 \text{ kip}$$

$$A_y = 50 \text{ kip} - 20 \text{ kip} = 30 \text{ kip} \downarrow$$

$$\sum F_x = 0 = A_x$$



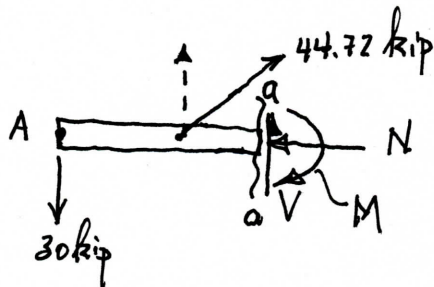
To find the tension in the cable consider point C:



$$\tan^{-1}(1/2) = 26.56^\circ$$

$$\frac{20 \text{ kip}}{\sin 18.43^\circ} = \frac{F_{CE}}{\sin(45^\circ)} \quad \text{from Law of Sines}$$

$$F_{CE} = 20 \text{ kip} \frac{\sin(45^\circ)}{\sin(18.43^\circ)} = 44.72 \text{ kip}$$



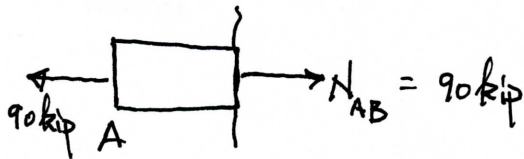
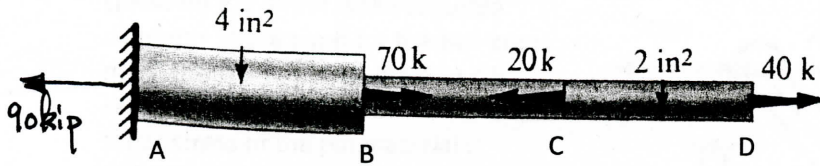
$$\sum F_x = 0 = 44.72 \text{ kip} \cos 26.56^\circ - N \rightarrow N = 40 \text{ kip (C)}$$

$$\sum F_y = 0 = -30 \text{ kip} + 44.72 \sin(26.56^\circ) + V \rightarrow V = 10 \text{ kip as shown}$$

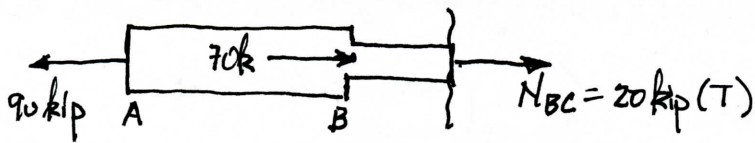
$$\sum M_A = 0 = (44.72) \sin(26.56^\circ)(3') + (10 \text{ kip})(4') - M$$

$$M = 100 \text{ kip} \cdot \text{ft as shown}$$

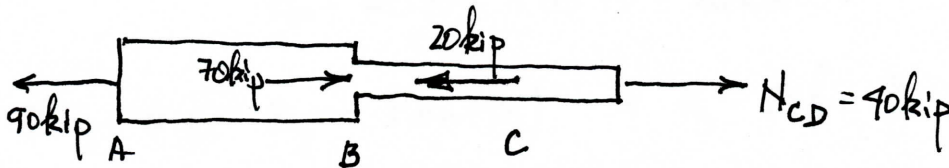
**Question #2.** For the segmented bar shown, calculate the average normal stress over the cross-sections in all of the segments AB, BC, and CD.



$$\bar{\sigma}_{AB} = \frac{N_{AB}}{4 \text{ in}^2} = \boxed{22.5 \text{ ksi (T)} = \bar{\sigma}_{AB}}$$

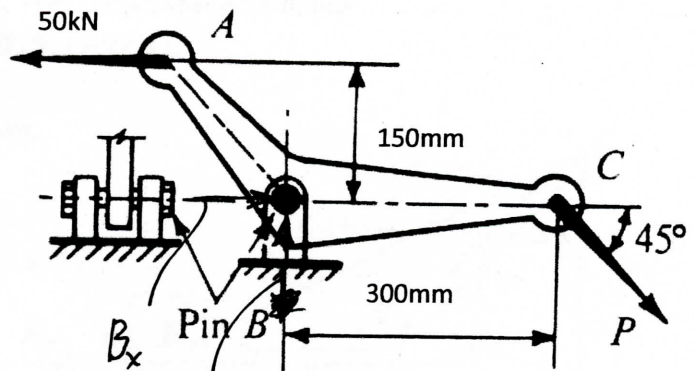


$$\bar{\sigma}_{BC} = \frac{N_{BC}}{2 \text{ in}^2} = \frac{20 \text{ kip}}{2 \text{ in}^2} = \boxed{10 \text{ ksi (T)} = \bar{\sigma}_{BC}}$$



$$\bar{\sigma}_{CD} = \frac{N_{CD}}{2 \text{ in}^2} = \boxed{20 \text{ ksi (T)}}$$

**Question #3.** What is the required diameter of the pin B for the bell-crank mechanism if the 50kN force at A is resisted by a force P at C? The ultimate shear stress of the pin material is 450 MPa, and a factor of safety of 3 is required.



$$\sum M_B = 0 = (50 \text{ kN})(150 \text{ mm}) - P \sin(45^\circ)(300 \text{ mm}) \quad B_y$$

$$P = 50 \text{ kN} \left( \frac{150 \text{ mm}}{300 \text{ mm}} \right) \left( \frac{1}{\sin 45^\circ} \right) = 35.36 \text{ kN}$$

$$\sum F_x = 0 = -50 \text{ kN} + B_x + P \cos 45^\circ$$

$$B_x = 50 \text{ kN} - P \cos 45^\circ = 25 \text{ kN} \rightarrow$$

$$\sum F_y = 0 = B_y - P \sin(45^\circ) \rightarrow B_y = 25 \text{ kN} \uparrow$$

$$R_B = \sqrt{B_x^2 + B_y^2} = 35.36 \text{ kN}$$

The pin at B is double-sheared.  $\therefore V_B = R_B/2 = 17.68 \text{ kN}$

$$\tau_{all} = \frac{\tau_{ult}}{FS} = \frac{450 \text{ MPa}}{3} = 150 \text{ MPa} = 150 \text{ N/mm}^2$$

$$\tau_B = \frac{V_B}{\frac{\pi d_B^2}{4}} = \tau_{all} \rightarrow d_B = \sqrt{\frac{4V_B}{\pi \tau_{all}}} = \sqrt{\frac{4 \times 17,680 \text{ N}}{\pi \times 150 \text{ N/mm}^2}} = 12.25 \text{ mm}$$

$$\therefore d_B \geq 12.25 \text{ mm}$$