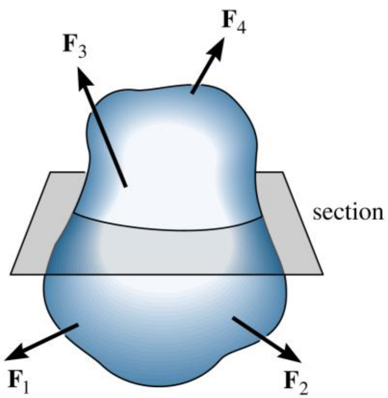
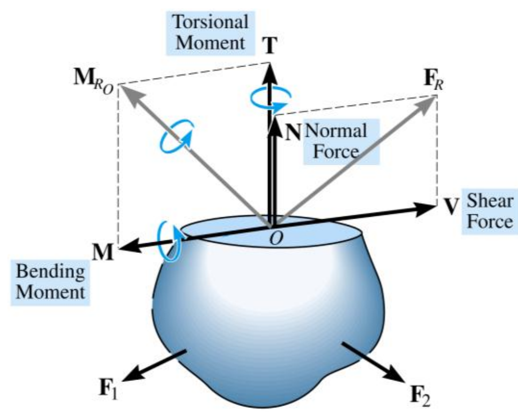
Period #2 : Average Normal Stresses

A. Review of Statics

When external loads are applied to a restrained deformable body, internal forces are generated to keep the body in equilibrium.

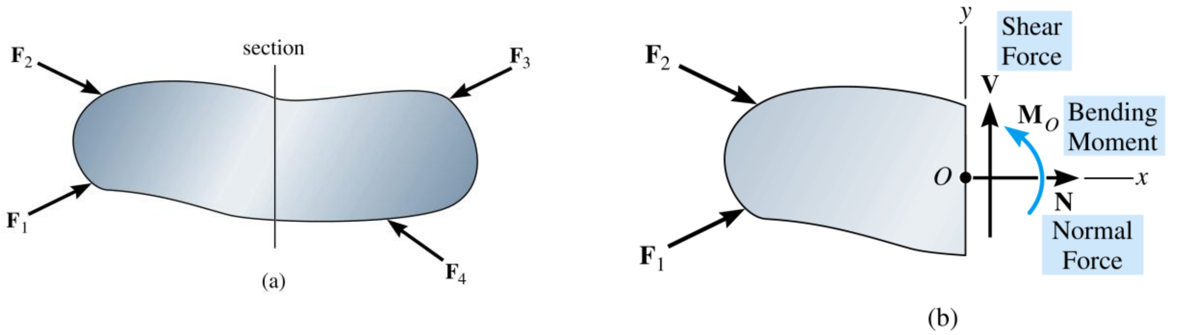
The internal forces in a body can be examined using methods of sectioning and principles of statics.





On a given section, the normal force **N** and the bending moment **M** give rise to normal stresses.

System of Co-Planar Forces



Summary: Using methods of statics we can determine the internal forces and moments.

B. Stress

On a section through a body, the internal forces and moments are created by stresses.

At a point on a section, stress is *force per unit area*.

Forces per unit area that act normally to the section are called normal stresses.
In this course, normal stresses are denoted by the symbol σ.

Forces per unit area that act tangentially to the section are called shear stresses.
Shear stresses in this course are denoted by the symbol τ.

Consider the body shown below with a cut section having a unit normal in the z-direction:

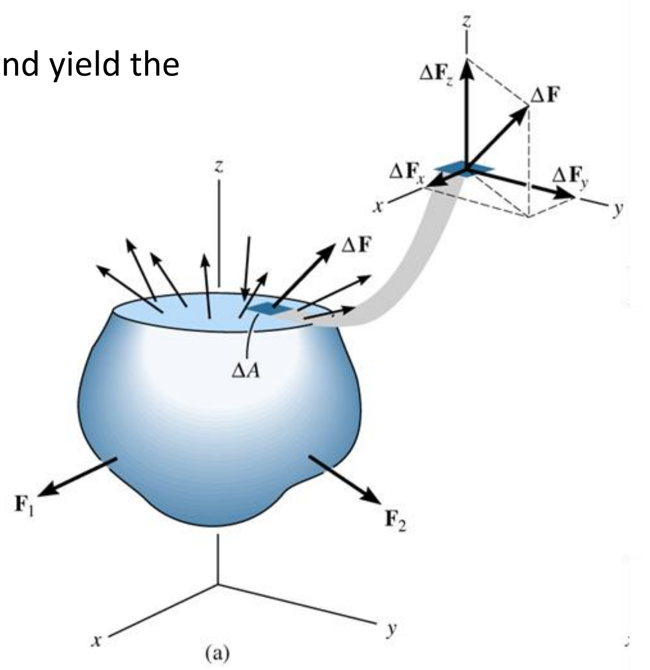
The force acting force on an infinitesimal area ΔA, is $\Delta \mathbf{F}$.

The z-component of force $\Delta \mathbf{F}$ acts normally to the section and yields the normal stress component σ_z

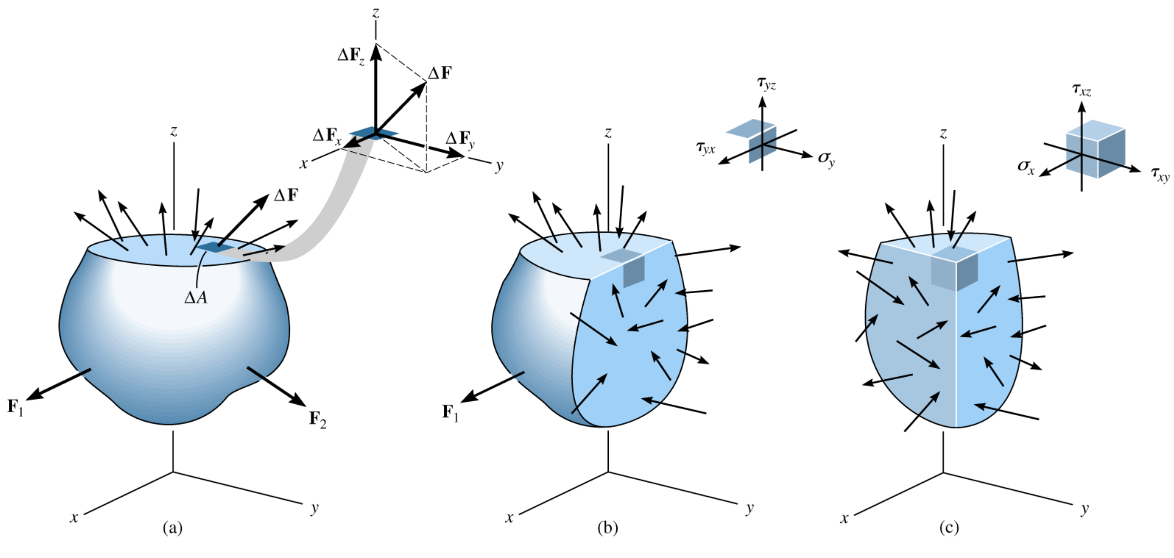
$$\sigma_z = \frac{\Delta F_z}{\Delta A}$$

The x- and y-components of force ΔF act tangentially to the section and yield the shear stress components τ_{zx} and τ_{zy}, respectively.

$$\tau_{zx} = \frac{\Delta F_x}{\Delta A} \qquad \tau_{zy} = \frac{\Delta F_y}{\Delta A}$$



More generally, the figure below shows the stress components at the same point when sections orthogonal to the y- and x-axes are also considered.



For an infinitesimal area ΔA orthogonal to the y-axis with force components ΔF_y, ΔF_x, and ΔF_z acting upon it:

$$\sigma_y = \frac{\Delta F_y}{\Delta A} \qquad \tau_{yx} = \frac{\Delta F_x}{\Delta A} \qquad \tau_{yz} = \frac{\Delta F_z}{\Delta A}$$

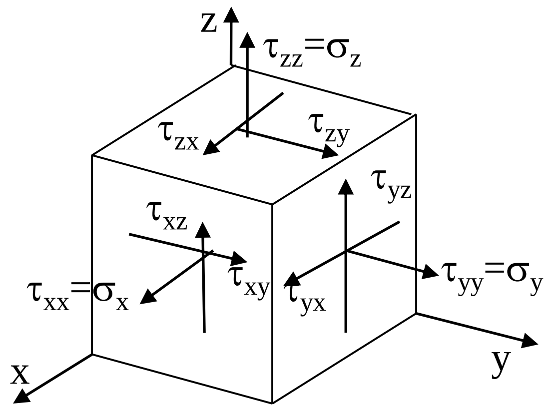
For an infinitesimal area ΔA orthogonal to the x-axis with force components ΔF_x, ΔF_y, and ΔF_z acting upon it:

$$\sigma_x = \frac{\Delta F_x}{\Delta A} \qquad \tau_{xy} = \frac{\Delta F_y}{\Delta A} \qquad \tau_{xz} = \frac{\Delta F_z}{\Delta A}$$

The common definition of stress is force per unit area.

To be precise, we need to be very specific about the directionality of the forces and the orientation of the area on which the forces are acting. This is done by representing stress as a rank-2 tensor as follows:

$$\boldsymbol{\tau} = \begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix} = \begin{bmatrix} \sigma_{x} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{y} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{z} \end{bmatrix}$$



The diagonal entries in the stress tensor are said to be **normal stresses** because the associated force component acts normal to the plane.

The off-diagonal entries in the stress tensor are called **shear stresses**, because the associated force components act parallel to the plane

A few points about stress:

The dimensions of stress are FL^{-2} where F denotes a unit of force and L denotes a unit of length. Examples are:

N/m^2 where N is a newton and m denotes a meter.
One newton per square meter is called 1 pascal or 1 Pa.
lb/in^2 or pounds per square inch (psi).

Normal stress, in this course, is taken positive in tension and negative in compression.

Stresses are point-wise quantities that generally vary over a section in a body.

Sometimes, we are interested in the average stresses acting on a given section. The average normal stress on a given section is defined as follows:

$$\bar{\sigma} = \frac{\int \sigma \, dA}{A}$$

Though we should, we will not always use the overbar to represent average normal stresses.

C. Average Normal Stress in Axially Loaded Members

Axial members by definition possess a longitudinal axis.

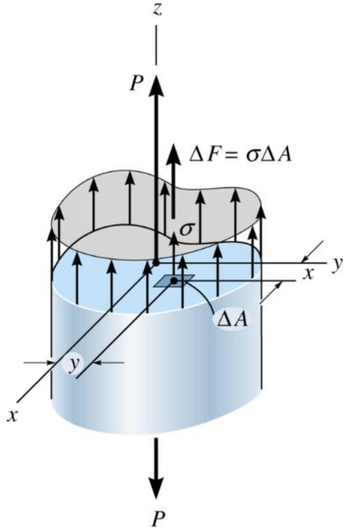
Cross-sections of axial members have normals that are parallel to the longitudinal axis.

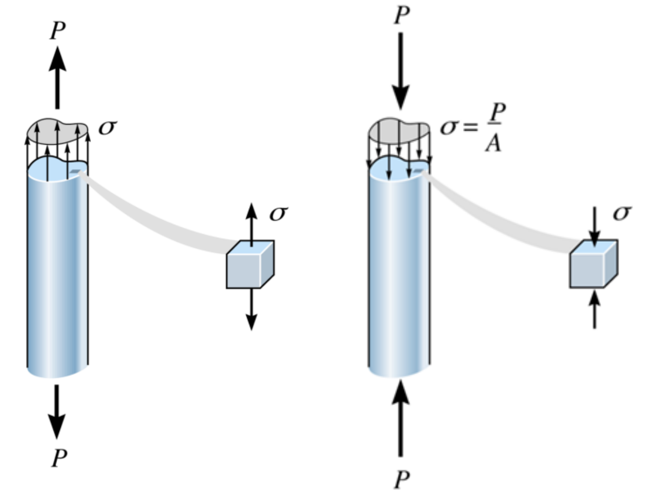
Purely axially loaded members undergo strictly axial loading. The resultant normal force at any cross-section passes through the centroid of that section.

•The **average normal stress** at a cross-section of an axially loaded member is:

$$\sigma = \frac{P}{A}$$

where P is the axial force at the given cross-section, and A is the area of the given cross-section.



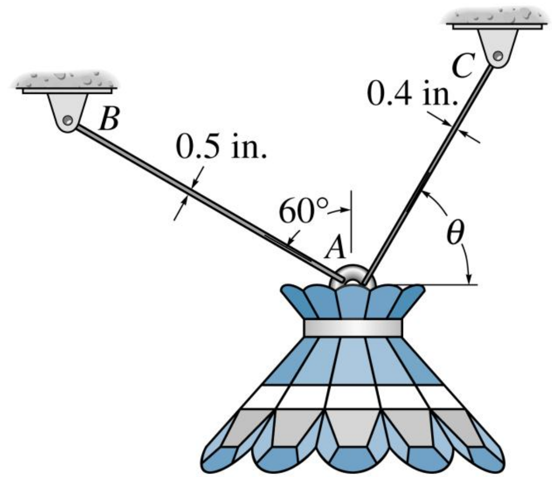


Tension

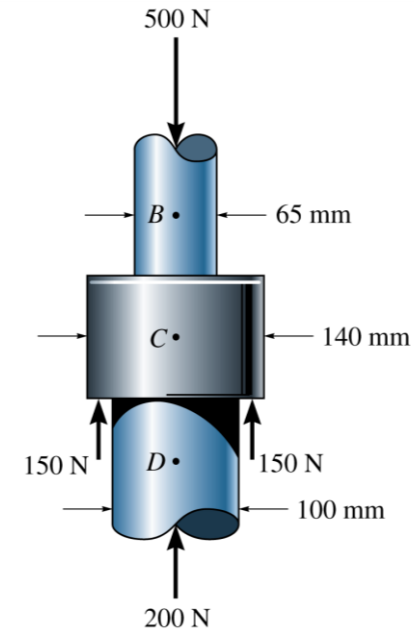
Compression

D. Example Problems

Example 2.1 The 50-lb lamp is supported by the two-rods shown. Determine the angle of orientation q of member AC such that the average normal stress in rod AC is twice the average normal stress in rod AB. What is the magnitude of stress in each rod?



Example 2.2 The thrust bearing is subjected to the loads shown. Determine the average normal stress developed on cross sections through points B, C, and D.



Example 2.3 The joint shown is subjected to the axial member force of 6 kip. Determine the average normal stress acting on sections AB and BC. Assume the member is *smooth* and is 2 inches thick.