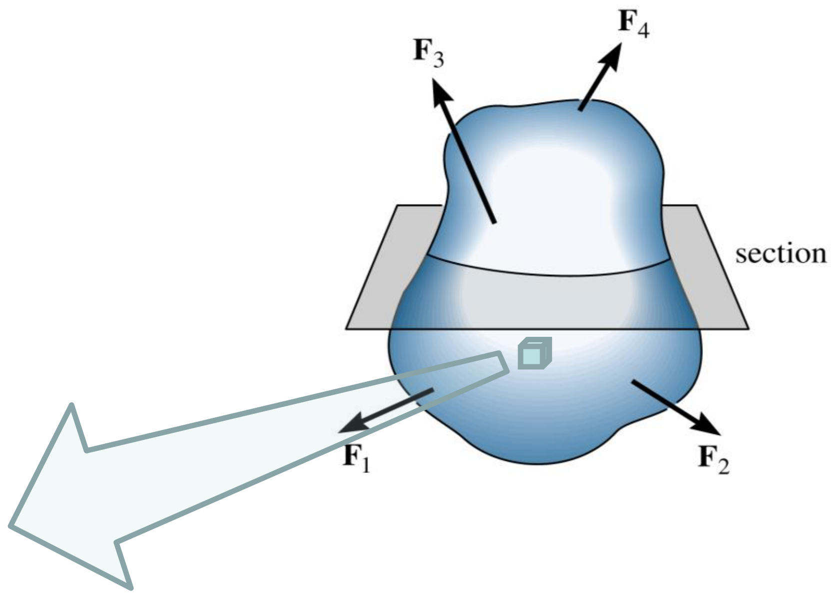
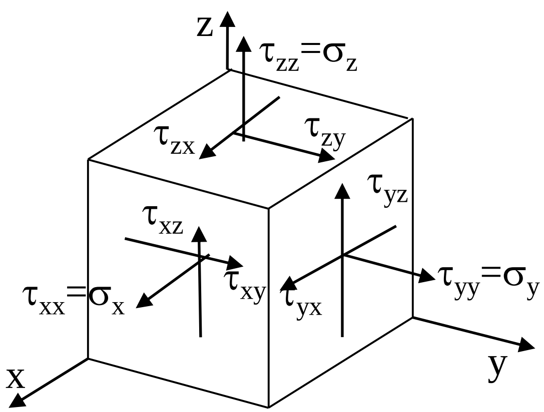
Period #3 : Average Shear Stresses

A. Review of Stresses Thus Far

At a point in a body, the state of stress is generally represented with a stress tensor.





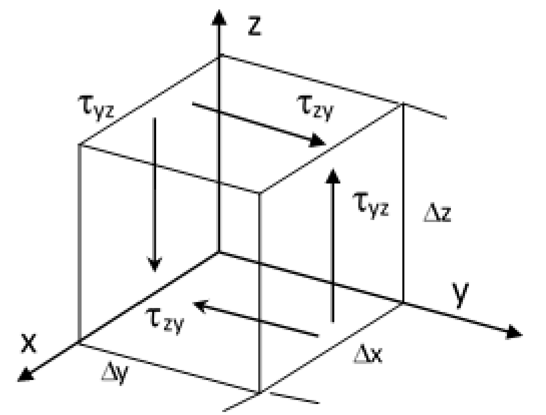
$$\boldsymbol{\tau} = \begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$

B. Symmetry of the Stress Tensor

The stress tensor is generally symmetric, because:

$$\tau_{xy} = \tau_{yx}, \quad \tau_{xz} = \tau_{zx}, \text{ and } \quad \tau_{yz} = \tau_{zy}$$

We can demonstrate this by showing that $\tau_{zy}=\tau_{yz}$. The demonstrations that $\tau_{xy}=\tau_{yx}$ and $\tau_{xz}=\tau_{zx}$ are similar.



To show that $\tau_{zy}=\tau_{yz}$ consider the infinitesimal cube at right. The area of faces with normal along the y-axis are: $\Delta A_y=(\Delta x)(\Delta z)$. Similarly, the faces with normal in the z-direction have $\Delta A_z=(\Delta x)(\Delta y)$.

Now, consider moment equilibrium of the cube about the x-axis.

$$\sum M_x = 0 = \tau_{yz}\Delta A_y \Delta y - \tau_{zy}\Delta A_z \Delta z$$

$$\tau_{yz}\Delta A_y \Delta y = \tau_{zy}\Delta A_z \Delta z$$

$$\tau_{yz}\Delta x \Delta z \Delta y = \tau_{zy}\Delta x \Delta y \Delta z$$

$$\tau_{yz} = \tau_{zy}$$

This basic fact concerning symmetry of shear stresses will be utilized frequently in this course.

C. Average Shear Stresses

The shear stress will typically vary over a section in a body.

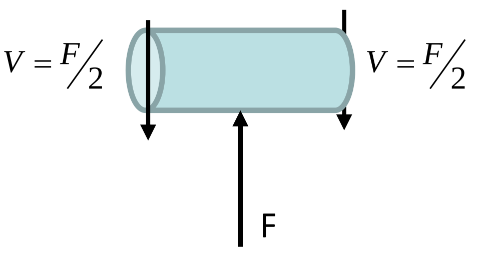
However, the <u>average</u> shear stress on a section is calculated by taking the spatial average:

$$\bar{\tau} = \frac{\int_A \tau \, dA}{\int_A dA}$$

If we know only the resultant shear force V acting on a section with area A, the average shear stress on that section would be:

$$\bar{\tau} = \frac{V}{A}$$

Later in the course we will solve for shear stress distributions on cross-sections of shafts and beams, but for now, our interest is simply average shear stresses.



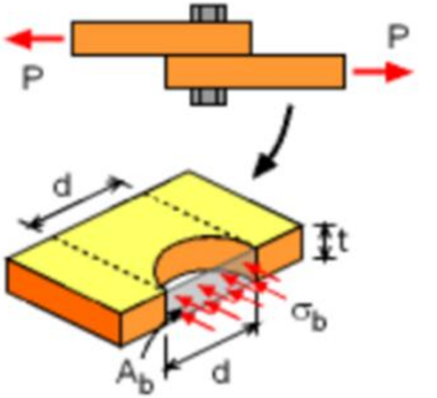
D. Average Bearing Stresses

When two distinct objects are in contact with one other, there can be compressive normal stresses (or bearing stresses) on the interface or surface that separates them.

The magnitude of the average bearing stresses will be the magnitude of the net bearing force divided by the contact area orthogonal to that force.

$$\bar{\sigma}_b = \frac{F}{A_n}$$ where A_n is the area orthogonal to F.

For example in the connection shown at right a contact force P exists between a bold of diameter d and the plate of thickness t.



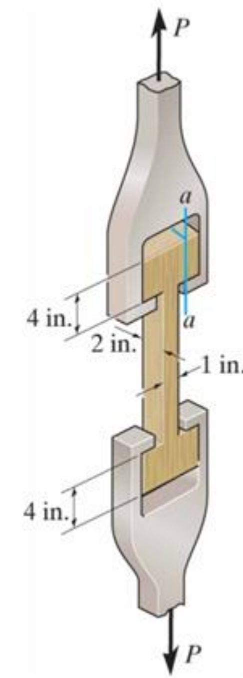
Bearing Stress Due to a Bolt

The normal contact area here is d*t, and the average bearing stress would be:

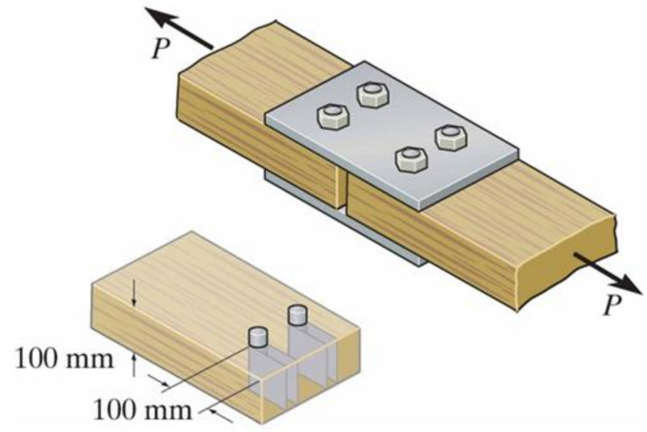
$$\bar{\sigma}_b = \frac{P}{d*t}$$

E. Example Problems

Example 3.1 During the tension test, the wooden specimen is subjected to an average normal stress of 2ksi. Determine the axial force **P** applied to the specimen. Also, find the average shear stress along section *a-a* of the specimen.



Example 3.2 The average shear stress in each of the 6-mm diameter bolts and along each of the four shaded shear planes in the wood is not allowed to exceed 80 MPa and 500 kPa, respectively. Determine the maximum axial force P that can be applied to the joint. For that axial force P, find the average bearing stress between the bolts and the wood.



Example 3.3 The open square butt joint is used to transmit a force of 50 kip from one plate to the other. Determine the average normal and average shear stress components that this loading creates on the face of the weld, section AB.