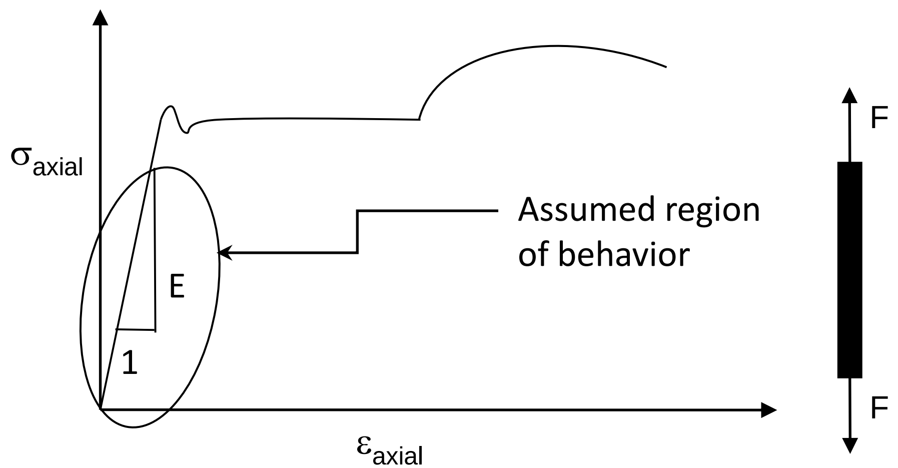
Period #7: Axially Loaded Members

A. Context

For now, we confine our attention to axial members in tension or compression in the linear, elastic regime of behavior.

Thus it is assumed that the magnitudes of axial stresses in the members are less than the material yield stresses, and the magnitudes of axial strains are less than the material yield strains.



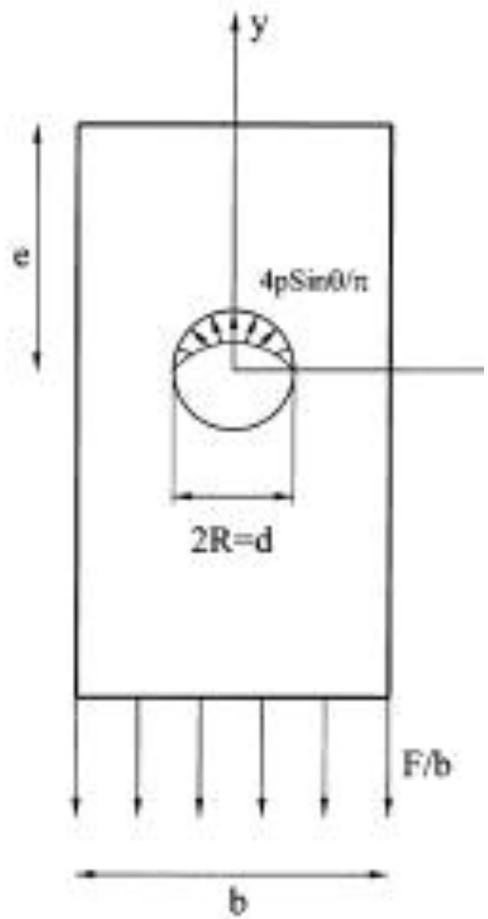
$$\bar{\sigma}_{axial} = F/A$$

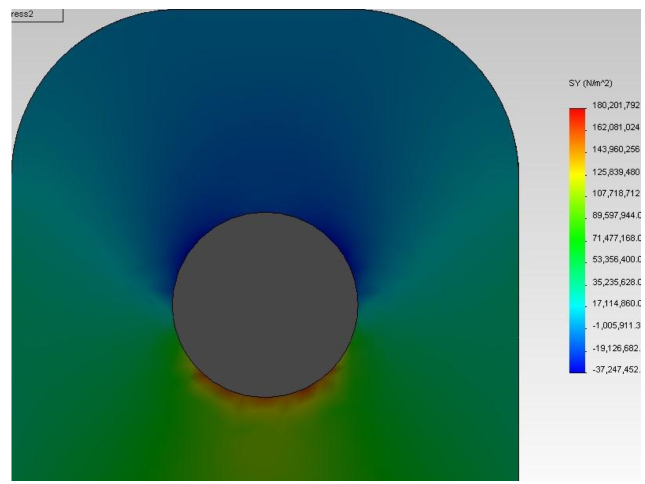
$$\varepsilon_{axial} = \bar{\sigma}_{axial}/E$$

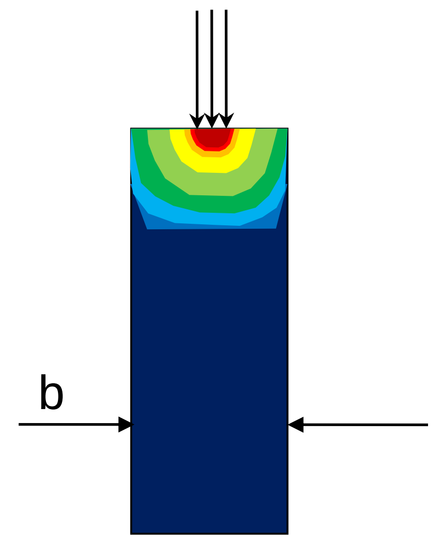
$$\varepsilon_{lateral} = -\nu\varepsilon_{axial}$$

At the point (or area) where loads are actually applied to axial members, the stresses are not uniform. As one moves away from the area though, the stresses even out pretty quickly. This behavior is known as St. Venant's effect.

In subsequent courses you might consider the concentrated stresses around these connections, but for now, we will neglect these regions.

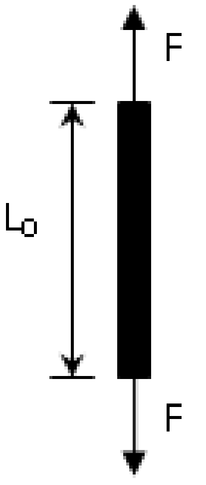






B. Axial Stresses, Strains, and Deformations

1. Constant Area and Force



In cases such as this, the axial stress and strain will be essentially uniform throughout the member

$$\sigma_{axial} = F/A$$

$$\varepsilon_{axial} = \frac{\sigma_{axial}}{E} = \frac{F}{AE} = \frac{\Delta L}{L_0}$$

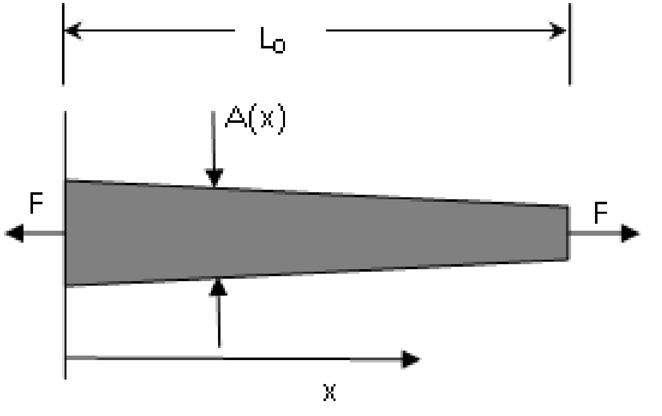
$$\boxed{\therefore \Delta L = \frac{FL_0}{AE}}$$

2. Constant Force but Variable Area

$$\sigma(x) = \frac{F}{A(x)}$$

$$\varepsilon(x) = \frac{\sigma(x)}{E(x)} = \frac{F}{A(x)E(x)}$$

$$\Delta L = \int_0^{L_0} \varepsilon(x)dx = \int_0^{L_0} \frac{F}{A(x)E(x)}dx$$

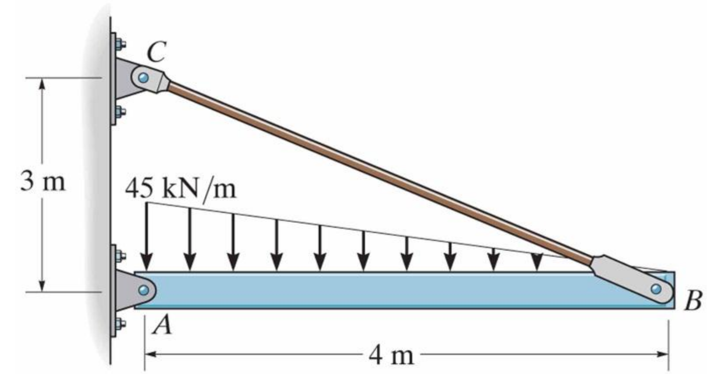


3. Variable Force and Area

This case is treated in Example #7.3 below.

C. Examples

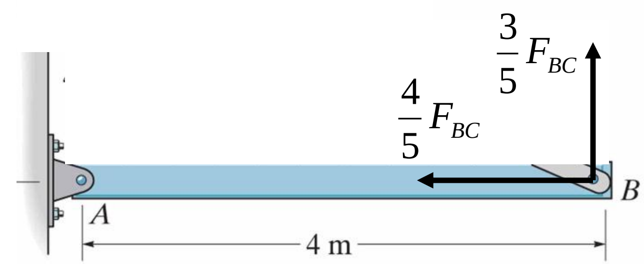
Example #7.1: The rigid beam AB is subjected to the loading and support conditions shown. Rod BC is A36 steel with a diameter of 25.4mm. Determine the rotation of beam AB about the pinned support at A.



Solution:

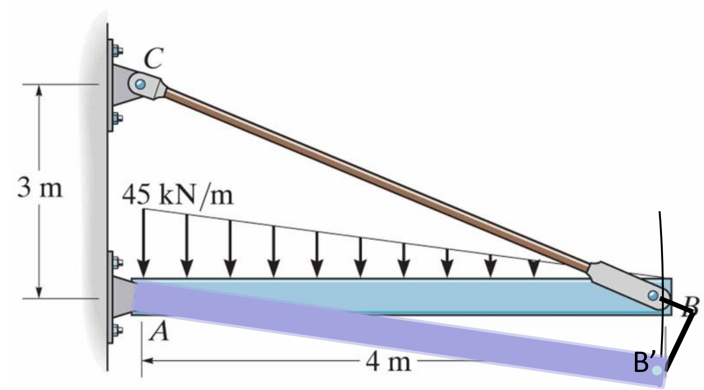
$$\sum M_A = 0 = (-90kN)(2m) + \left(\frac{3}{5}F_{BC}\right)(4m)$$

$$F_{BC} = 75kN$$



$$\Delta L_{BC} = \frac{(F_{BC})(L_{BC})}{AE} = \frac{(75kN)(5m)}{\left(\frac{\pi}{4}\right)(.0254m)^2\left(200\cdot 10^6 kN\cdot m^{-2}\right)}$$

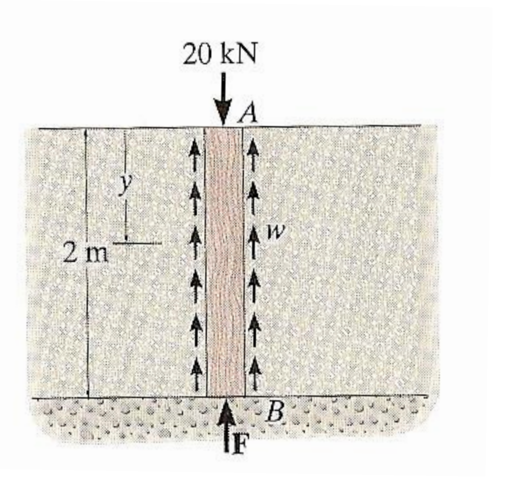
$$= 0.0037m = 3.7mm$$



Observe that while the rod BC extends by 3.7mm, it also rotates about point C. Point B moves to point B', moving 3.7mm parallel to BC, and then 3.7mm *(4/3) perpendicular to BC. The resultant motion is the vertical displacement BB'=3.7mm*(5/3)=6.17mm downward.

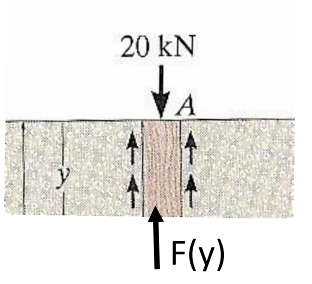
The beam AB rotates by θ=(6.17/4000)=.00154rad or .0884 degrees.

Example #7.2: The post is made of Douglas fir and has a diameter of 60mm. If it is subjected to the load of 20kN and the soil provides a frictional resistance that is uniformly distributed along its sides of w=3kN/m, determine the bearing force F at the bottom needed for equilibrium. Also, what is the displacement of the top of the post A with respect to its bottom B? Neglect the weight of the post.



Solution:

From the textbook, E=13.1 GPa for Douglas fir.



$$F(y) = 20kN - w*y$$

$$F(2m) = 20kN - 3kN/m*2m$$

$$= 14\text{kN} \ \ (C)$$

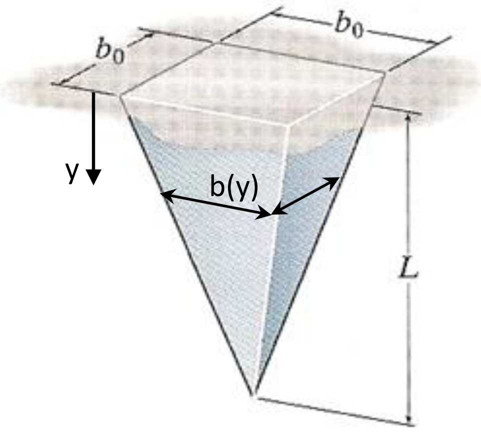
$$\sigma(y) = \frac{F(y)}{A} = \frac{(20-3*y)kN}{A}$$

$$\varepsilon(y) = \frac{\sigma(y)}{AE} = \frac{(20-3y)kN}{AE}$$

$$\Delta L = \int_0^L \varepsilon(y)dy = \frac{1}{AE}\int_0^L F(y)dy = \frac{1}{AE}\int_0^L (20-3y)dy = \frac{1}{AE}\left((20kN)L - \frac{3kNm^{-1}}{2}L^2\right)$$

$$= \frac{34\text{kN}\cdot\text{m}}{\text{AE}} = \frac{34\text{kN}\cdot\text{m}}{\frac{\pi}{4}(.06\text{m})^2(13.1\cdot 10^6 kN\cdot m^{-2})} = 9.18^{-4}m = 0.918mm$$

Example #7.3: A casting is made of a material that has a specific weight γ and modulus of elasticity E. If it is formed into a pyramid having the dimensions shown, determine how far its end is displaced due to gravity when it is suspended in the vertical position as shown.



Solution:

Observe that a distance y down from its top, the pyramid has a side length b(y)

$$b(y) = b_o\left(1-\frac{y}{L}\right)$$

$$A(y) = b^2(y) = b_o^2\left(1-\frac{y}{L}\right)^2 = A_o\left(1-\frac{y}{L}\right)^2$$

The volume of the pyramid <u>beneath</u> a level y would be:

$$V(y) = \frac{1}{3}A(y)h(y) = \frac{1}{3}A_o\left(1-\frac{y}{L}\right)^2(L-y) = \frac{1}{3}A_oL\left(1-\frac{y}{L}\right)^3$$

The corresponding weight beneath y would be:

$$W(y) = \gamma V(y) = \frac{\gamma}{3}A_oL\left(1-\frac{y}{L}\right)^3$$

The average axial stress at level y would be:

$$\bar{\sigma}(y) = \frac{W(y)}{A(y)} = \frac{\frac{\gamma}{3}A_oL\left(1-\frac{y}{L}\right)^3}{A_o\left(1-\frac{y}{L}\right)^2} = \frac{\gamma L}{3}\left(1-\frac{y}{L}\right)$$

strain ε(y):

$$\varepsilon(y) = \frac{\gamma L}{3E}\left(1-\frac{y}{L}\right) = \frac{\gamma}{3E}(L-y)$$

Tip displacement:

$$\boxed{\Delta L = \int_0^L \varepsilon\, dy = \frac{\gamma L^2}{6E}}$$