Period #10: Calculation of Twist in Shafts

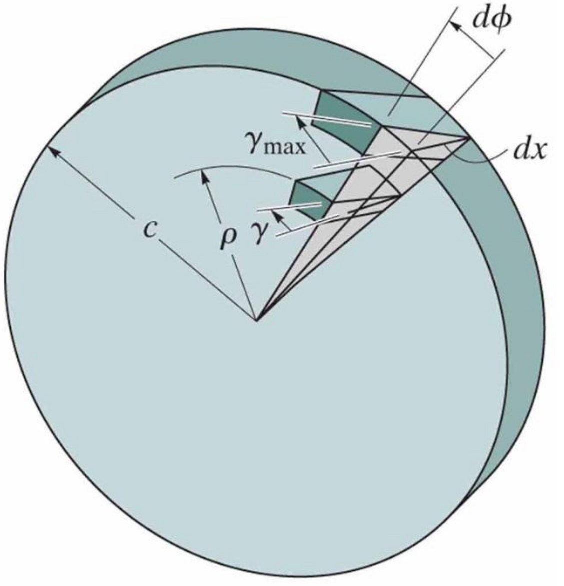
A. Review

1. Section properties

Polar moment of inertia: $J = \int_A \rho^2 dA$

Circular cross-section: $J = \frac{\pi}{2} c^4 = \frac{\pi}{32} d^4$

Annular cross-section: $J = \frac{\pi}{2}\left(c_o^{\,4} - c_i^{\,4}\right) = \frac{\pi}{32}\left(d_o^{\,4} - d_i^{\,4}\right)$



The shear strain at points on the cross section increases linearly with ρ, i.e., $\gamma = (\rho/c)\gamma_{max}$.

2. Torsion formulae

$$\tau_{max} = \frac{Tc}{J} \qquad \tau = \frac{T\rho}{J}$$

$$\gamma_{max} = \frac{Tc}{GJ} = c\frac{d\phi}{dx}$$

$$d\phi = \frac{T}{GJ}dx$$

3. Power transmission

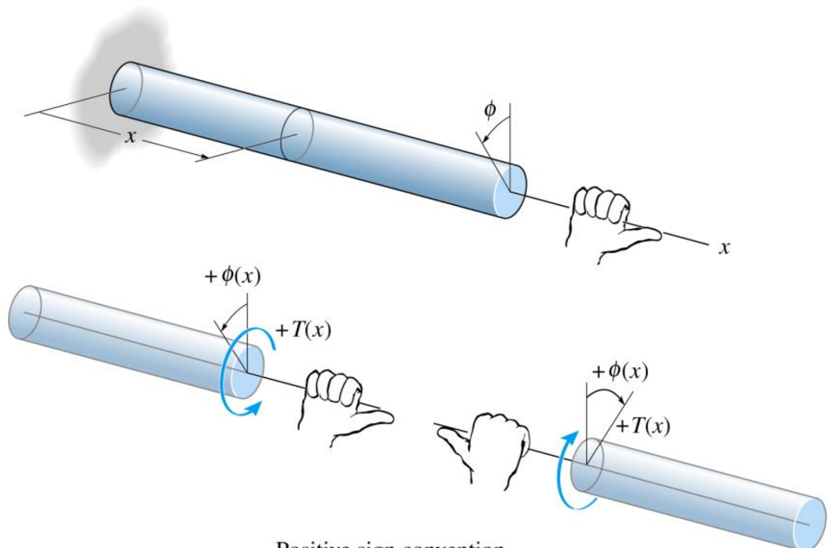
$$P = T\omega$$

B. Angle of Twist

1. Sign Convention

For both torque and angle of twist the sign convention is the same.

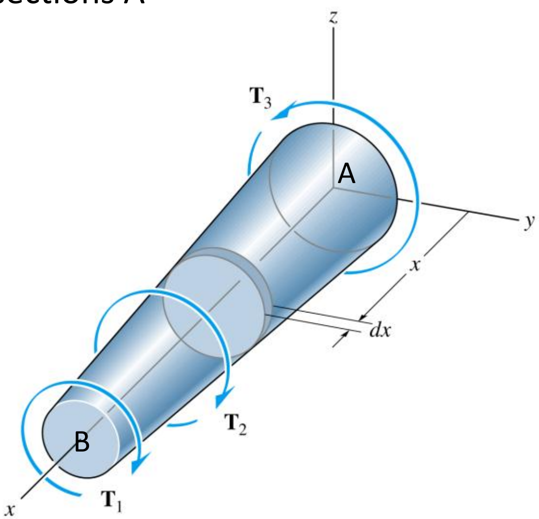
Using the right-hand-rule, if the vector resultant of the torque or twist points away from the cross-section, it is positive.



Positive sign convention
for T and ϕ

2. Relative twist angles

To calculate the change in angle of twist between two sections A and B on a shaft:



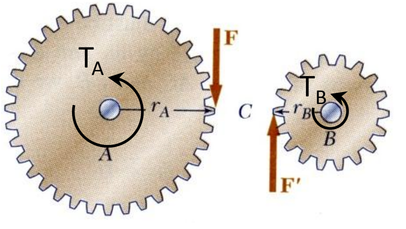
$$\phi_{B/A} = \int_A^B \frac{\mathrm{T}}{\mathrm{GJ}} dx$$

If T, G, and J are constant from A to B:

$$\phi_{B/A} = \frac{\mathrm{TL_{AB}}}{\mathrm{GJ}}$$

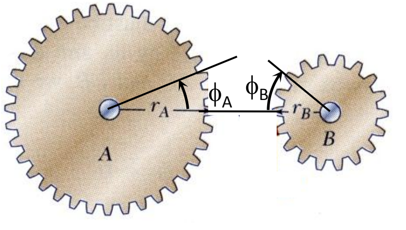
3. Treatment of Meshed Gears

The contact force between the two gears, F and F’ are equal and opposite.



Equilibrium of gear A: $F=T_A/r_A$
Equilibrium of gear B: $F'=T_B/r_B$
$F=F' \rightarrow T_B=(r_B/r_A)T_A$

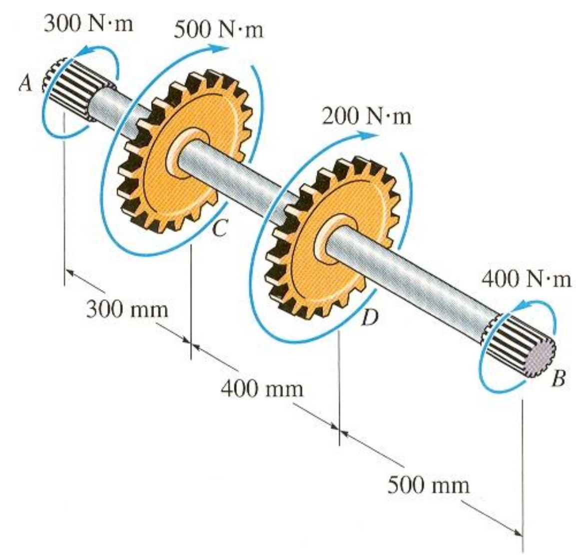
Compatibility of gear motion:



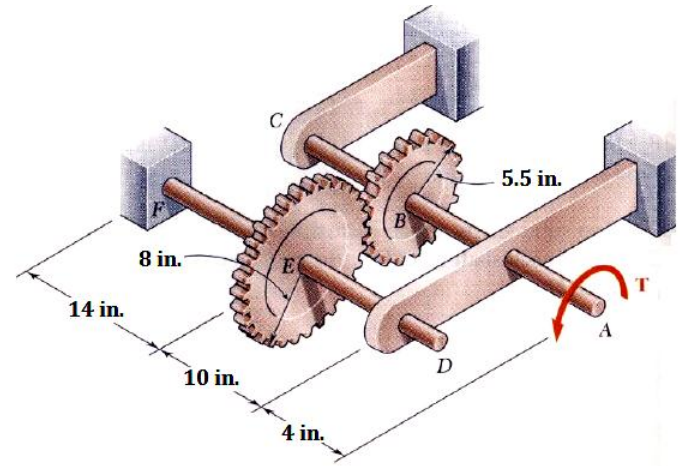
$$\phi_A r_A = -r_B \phi_B \;\rightarrow \phi_{\mathrm{B}} = -\frac{r_A}{r_B}\phi_A$$

C. Example Problems

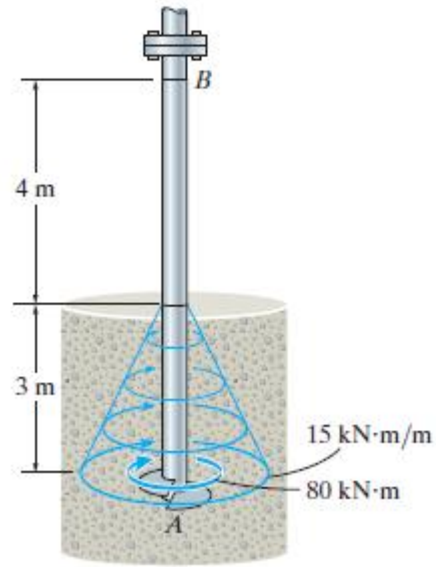
Example 10.1: The splined ends and gears attached to the A-36 steel shaft are subjected to the torques shown. Determine the angle of twist of gear *C* with respect to gear *D*. The shaft has a diameter of 40mm.



Example 10.2: Two shafts, each ¾ in diameter are in contact via the gears shown. The shaft at F is fixed in place and has a shear modulus $G=13.5x10^6$ psi. If a torque of T = 3 kip·in is applied at A determine the twist angle of the shaft at A.



Example 10.3: The A992 steel post is "drilled" at constant angular speed into the soil using the rotary installer. If the post has an inner diameter of 200 mm and an outer diameter of 225 mm, determine the relative angle of twist of end *A* of the post with respect to end *B* when the post reaches the depth indicated. Due to soil friction, assume the torque along the post varies linearly as shown, and a concentrated torque of 80 kN-m acts at the bit.



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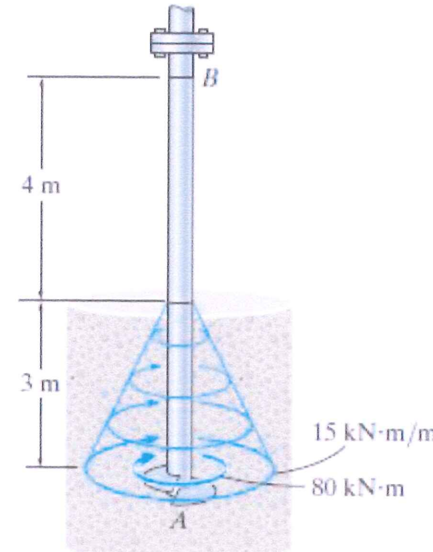
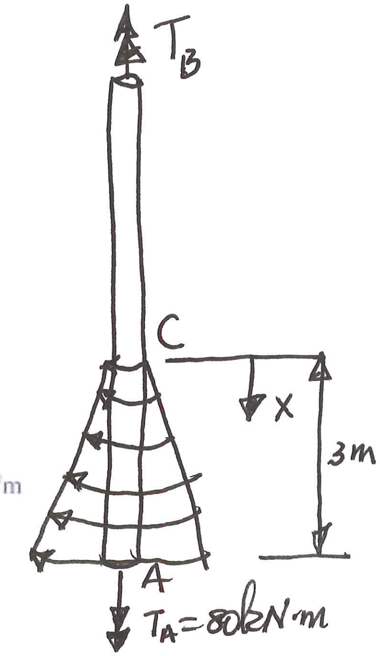
$G = 75\,GPa = 75 \cdot 10^6\,kPa$





from B to C, $T(x) = +102.5\,kN\cdot m$

from C to A $T(x) = 102.5 - \int_0^x t(\xi)\,d\xi$

$= 102.5\,kN\cdot m - \int_0^x 5\xi\,d\xi$

$= 102.5 - \frac{5x^2}{2}$

$$\phi_{A/B} = \frac{T_{BC} L_{BC}}{JG} + \int_C^A \frac{102.5 - 5x^2/2}{GJ}\,dx$$

$$= \frac{410\,kN\cdot m^2}{JG} + \frac{307.5\,kN\cdot m^2}{JG} - \frac{22.5\,kN\cdot m^2}{JG} = \frac{695\,kN\cdot m^2}{JG} = .0981\,rad = 5.62^\circ = \phi_{A/B}$$

$$JG = \frac{\pi}{2}(.1125m^4 - .10m^4)(75\cdot 10^6\,kPa) = 7090\,kN\cdot m^2$$

distributed torque applied to shaft $t(x) = 5\frac{kN}{m}x$

$$T_A + \int_C^A t(x)\,dx = T_B$$

$$80\,kN\cdot m + \int_0^{3m} \left(5\frac{kN}{m}\right) x\,dx = T_B$$

$$T_B = 102.5\,kN\cdot m$$