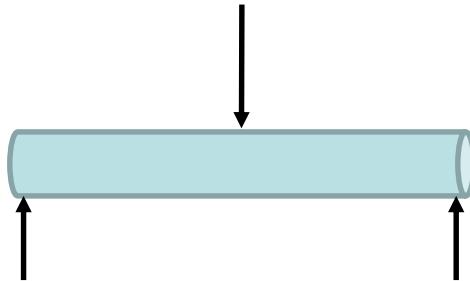# Period #12:  Bending Behavior

## A. Next Topic

Chapter 4 dealt with axial loading of axial members (rods)

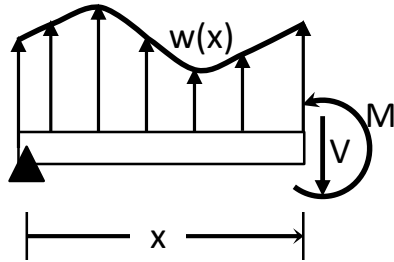Chapter 5 dealt with torsional loading of axial members (shafts)

Chapters 6 and 7 deals with transverse loading of axial members (beams).

Our goal in studying beams is to quantify the internal strains and stresses in members subjected to transverse loads.

## B. Beam Sign Conventions for Loads, Transverse Shears, and Bending Moments



Beams subjected to transverse loads w(x) will have internal shear forces V and bending moments M.

The sign conventions for w, V, and M are as shown below:



Positive external distributed load
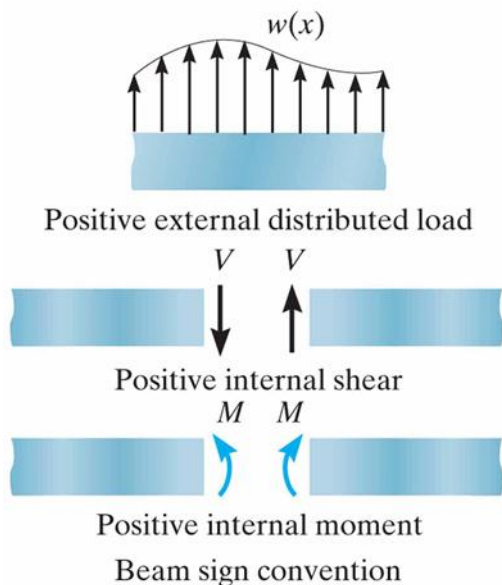
Loads acting upward are taken to be positive

Positive internal shear

Internal shears on cut faces that tend to cause counter-clockwise rotation are positive.

Positive internal moment

Beam sign convention

Internal bending moments that tend to cause compression of the top fibers of a beam are positive.

## C. Calculation of the Shear (V) and Moment (M) distributions

A shear diagram is a graphical representation of the transverse shear force V distribution in a beam.

A moment diagram is a graphical representation of the bending moment M distribution in a beam.

Given a loading that acts on specific beam with well-defined support conditions, it is necessary to be able to compute both V(x) and M(x).

1. **Relation between w and V**

Consider an infinitesimal beam segment of length dx:
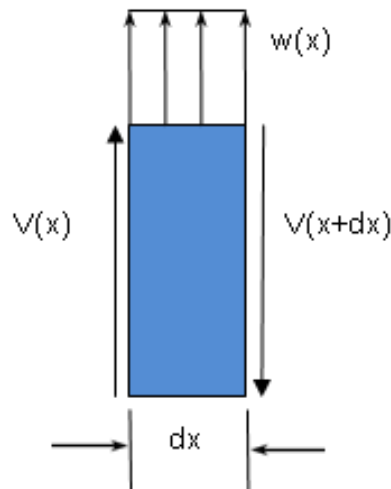
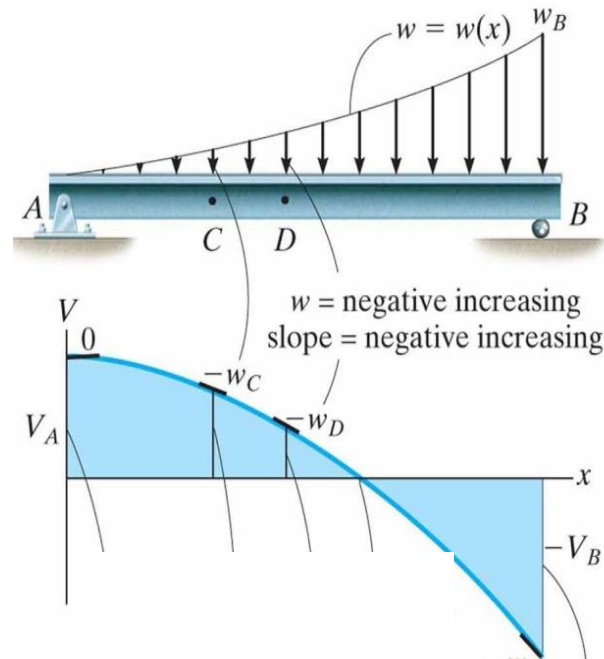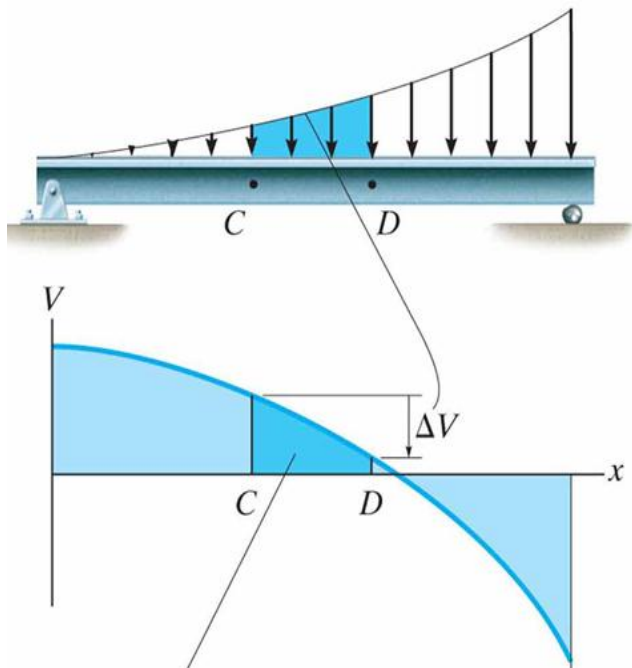$$\sum F_y = 0 \quad \text{[Neglect terms of order 2 or higher]}$$

$$V(x) + w(x)dx = V(x + dx)$$

$$\frac{V(x + dx) - V(x)}{dx} = w(x)$$

$$\frac{dV}{dx} = w(x) \quad \leftrightarrow \quad V_B - V_A = \int_A^B w(x)dx$$

In words, the shear in a beam is obtained by integrating the load function.

Alternatively, the slope of the shear diagram represents the intensity of the load function.



$w = w(x)$ $w_B$

$A$ $B$

$C$ $D$

$w$ = negative increasing
slope = negative increasing

$-w_C$

$-w_D$

$V_A$

$V$ $0$

$-V_B$

$\Delta V$

$V$

$C$ $D$

$x$

When a discrete load acts on a beam, the shear diagram increases or decreases with that load.

## 2. Relation Between V and M

Consider an infinitesimal segment of a beam dx:

$$\sum M = 0 \quad \text{[Neglect all terms of order 2 or higher]}$$

$$-M(x) + M(x+dx) - V(x)dx = 0$$

$$\frac{M(x+dx) - M(x)}{dx} = V(x)$$
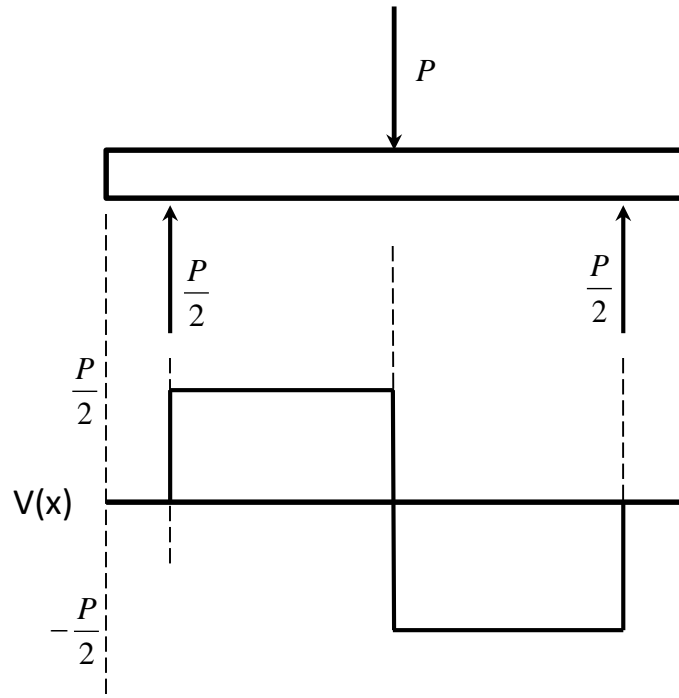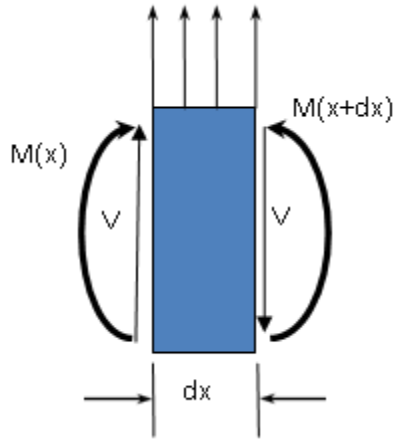
$$\frac{dM}{dx} = V(x) \quad \leftrightarrow \quad M_B - M_A = \int_A^B V(x)dx$$

In words, the moment in a beam is obtained by integrating the shear.

Alternatively, the slope of moment diagram at each point represents the shear at each point
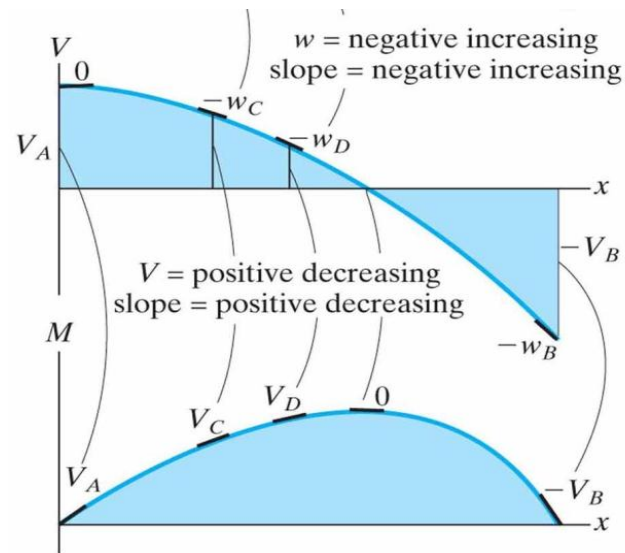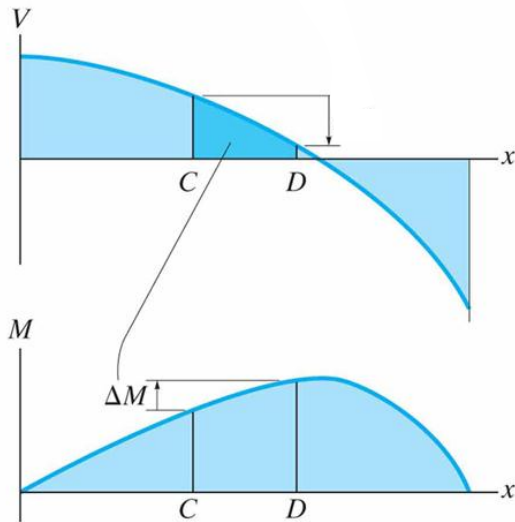
When a concentrated clockwise moment is applied to a beam, the moment distribution increases with that moment.
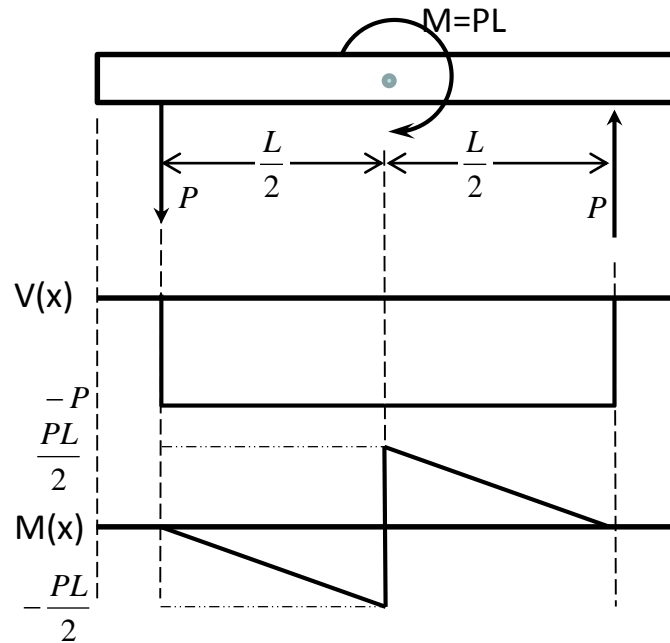
## D. Examples

**Example 12.1**  Draw the shear and moment diagrams for the beam

**Example 12.2.** Express the internal shear and moment in terms of *x* and then draw the shear and moment diagrams.

**Example 12.2.** Express the internal shear and moment in terms of $x$ and then draw the shear and moment diagrams.

$$w(x) = -\frac{2W_o}{L}\left(x - \frac{L}{2}\right)$$

$$m = \frac{W_o}{L/2} = \frac{2W_o}{L}$$

$$w(x') = -2W_o\frac{x'}{L}$$
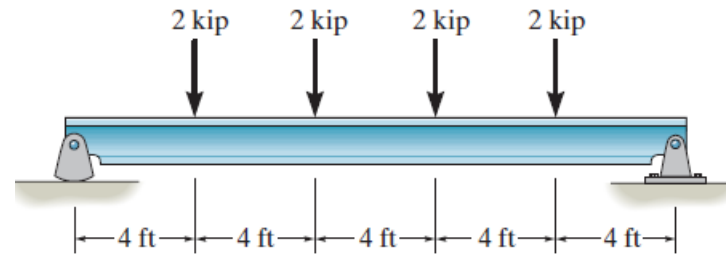


$$R_A + R_B = \frac{1}{2}W_o\frac{L}{2} = \frac{W_oL}{4}$$

$$\circlearrowleft + \Sigma M_A = 0 = R_B L - \left(\frac{W_oL}{4}\right)\left(\frac{5L}{6}\right)$$

$$R_B = \frac{5}{24}W_oL$$

$$R_A = \frac{1}{24}W_oL$$

$$V(x) = \frac{W_oL}{24} + -\frac{2W_o}{L}\left(\frac{x^2}{2} - \frac{xL}{2}\right) \quad x > \frac{L}{2}$$

$$V(x') = \frac{W_oL}{24} - \frac{W_o x'^2}{L}$$

$$V = 0 \text{ at } x' = \frac{L}{\sqrt{24}} = 0.204L = x'^* \text{ (location of max moment)}$$

$$M(x'^*) = M_{max} = \frac{W_oL^2}{48} + \frac{W_oL}{24}(.204L) - \frac{W_o}{3L}(.204L)^3 = \boxed{0.0265\,W_oL^2 = M_{max}}$$

$$V(x') = \frac{W_oL}{24} - \frac{W_o x'^2}{L}$$

$$M(x') = \frac{W_oL^2}{48} + \frac{W_oLx'}{24} - \frac{W_o x'^3}{3L}$$